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CONTENTS

DEVELOPMENT OF PERIODIC LOAN REPAYMENT MODELS CONSIDERING RHYTHMIC SKIPS

Abdullah EROGLU, PhD[*]
Mehmet Levent ERDAS, PhD[**]

Abstract

The notion of loan repayments rest on the principle that present value of sum of the instalments are equal to present value of the loan total. In a standard loan repayment plan, periodic instalments are set to a fixed amount. Besides, loan repayment plans with geometrically or arithmetically increasing periodic amounts can also be found at mathematics of finance textbooks. Beyond that models for loan repayments with skips deal with types of loans in that some periods are predetermined to pass by without making any instalment. Payment skips in some periods have been requested by some clients as expenses in some months rise considerably. In this study, general formulas are derived under the assumptions that periodic loan repayments adjust with arithmetic gradient series and interrupt with rhythmic non-payment periods. Later, by setting the arithmetic change to zero, a general formula for loan repayment with equal periodic instalments that also has rhythmic skips has been derived. Same numerical examples with solutions are presented for the developed models. As a result numerical examples have been used in order to show the validity of the models.

Keywords: Loan Amortization, Periodic Payments, Skip Periods, Rhythmic Skips

JEL Classification: G12, G19, G21

1. Introduction

Engineering economics plays a significant role in decision sciences (Blank and Tarquin, 2005). The cash flows, time value of

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money and interest rates are the most substantial research area in mathematical finance (Aydemir and Eroğlu, 2014: 95; Parvez, 2005). It is conventional that bank loans are repaid in equal periodical instalment amounts. It is essential that financial institutions offer alternative loan repayment plans that fit individuals income flow, which may decrease and increase along the year depending on the months or seasons. The notion of loan amortization rests on the principle that present value of a loan is equal to sum of periodic payments. The problems with payments of a debt stand with equality of the debt's present value and the sum of the present values (Shao and Shoa, 1998). In addition, arithmetic and geometric change models are available in financial mathematics textbooks. In mathematical finance textbooks general formulas are available for the cases where periodic payments are kept equal or are set to change in geometric or arithmetic sequences along the term (Park, 1997: 55).

The loan repayment model with equal periodic payment is:

$$d = \frac{pr}{1 - R^{-n}} \tag{1}$$

The loan repayment model where the periodic payments change in geometric sequence along the term is:

$$d_k = dG^{k\text{-}1} \quad k = 1, 2, n \tag{2}$$

$$d = \begin{cases} \dfrac{p(r-g)}{1-i^n} & , \quad g \neq r \\ \dfrac{pR}{n} & , \quad g = r \end{cases} \tag{3}$$

The loan repayment model where the periodic payments change in arithmetic sequence along the term is (Eroglu, 2000):

$$d_k = d + (k-1)v \quad , \quad k = 1, \cdots, n \tag{4}$$

$$d = \frac{pr^2R^n + v\left[1 + nr - R^n\right]}{r\left(R^n - 1\right)} \tag{5}$$

Where:

d: instalment or periodic payment amount,

d_k: k^{th} period due payment amount,

n: number of periods,

p: loan amount,

r: periodic interest rate, $R = 1 + r$

g: proportionate change (geometric) in periodic payment amount and $G = 1 + g$, $i = GR^{-1}$

v: absolute change (arithmetic) in periodic payment amount.

After a certain time from it takes credit, the customer's ability to pay may change. Therefore the customer may want to determine own the amount of a certain number of instalments in the first few months. Above three models assume that periodic payments are made at the end of the period. Formato (1992) developed a model in which client asks for not to pay make payment in certain periods that he/she would choose, for instance due to vacation expenses. Formato's skip payment model extended to the case where periodic payments change in geometric sequence by Moon (1994) and to extend to the case where periodic payments change in geometric sequence by Eroglu and Karaoz (2002). General formulae for cases when outstanding instalments have regular or irregular parts and geometric or arithmetic changes from one period to another were first discussed by Eroglu (2000). Moreover, Eroglu (2001) developed skipped payment models where periodic payments changes in partially geometric and arithmetic sequences and skipped payments are arbitrary. In fact, all four studies the skip periods, in which instalments are not made, are chosen arbitrarily. Arbitrary skips in instalments impose that periodic payments halt with occasional non-payment periods which may also differ in duration (Eroglu and Ozturk, 2016). Here, at which cycles the instalments will be skipped in the models in question is selected randomly. In addition to the randomly skipped loan payment models, rhythmically skipped loan payment models were discussed by Eroglu et al. (2011), Eroglu and Ozdemir (2012) and Eroglu et al. (2013). On the other hand, a loan payment model of which the certain number of instalments in the first months is determined by the customer was explained with another approach by Eroğlu (2013a). This model was further developed by Eroglu (2013b), Eroglu et al. (2014) and Eroglu and Ozturk (2016) was addressed as a loan model in which the certain number of instalments is identify by the customer at the beginning and then the number of instalments indicates arithmetic changes, and as a loan payment model in which the number of instalments indicates the partial geometric change for periods created by the equal instalment cycles.

The purpose of this study is to provide an alternative repayment model for the debts owed to a credit institution or to a bank through developing a novel mathematical model in line with the demands of the consumers.

In this study, general formulas will be derived under two assumptions about periodic payments. First periodic payments adjust with arithmetic sequence. Second periodic payments halt with rhythmic skip intervals along the loan term. Later, by setting the arithmetic change to zero, a general formula for loan repayment with equal periodic instalments that also has rhythmic skips will be derived. Numerical examples will be used in order to show the practicalities of the models.

2. A model for loan repayment with periodic payments that has arithmetic change and rhythmic skips

Fixed payment models are commonly used model for the loan amortization payments that are given from credit institutions or banks. In current life, after a certain period from the repayments was started, the customer's payment facility could be involved from the acquisition of the variability. In this case, the customers want to change and make easy the instalment level in the amount of a certain number of beginning periods itself for considering under changes in income for the coming periods (Aydemir and Eroglu, 2014: 97).

In this model, loan amortization is made with periodic payments. We define two intervals, namely instalment interval and skip interval. Instalment interval is the time interval which only contains periods with successive periodic instalments without interruption or skipped. Similarly, skip interval is the time interval that only contains successive periods in which periodic payments are stopped. We assume that a loan repayment schedule or loan term has at least two instalment intervals. Thus a loan term starts and ends with instalment intervals, which all have same length or contains same number of payment periods. Skip intervals are located between the instalment intervals and are equal to each other as well in number of periods without pay. Thus, we assume that periodic payments halt with orderly non-payment periods which are also identical in duration, which makes the skips rhythmic rather than arbitrary. In contrary, arbitrary skips in instalments assumption impose that periodic payments halt with occasional non-payment periods which may also differ in duration. Rhythmic skip assumptions makes our proposed models differ from prior studies.

Following additional notations are used in our model.

f: Total number of payment periods in an instalment interval.

h: Total number of periods in a skip interval.

M_k: First period number of an instalment interval that comes right after k[th] skip interval.

L_{k+1}: Last period number of an instalment interval that comes right after k[th] skip interval.

d_{kj}: The total amount of periodic payments made at j[th] instalment interval that comes right after a skip interval.

s: Total number of skip intervals.

n: Duration of loan repayment schedule in number of periods.

We assume each instalment interval equal to each other. Same rule applies to skip intervals as well. Then, following expressions can be written:

$$M_k = k(f+h)+1, \quad k=0,....,s$$
$$L_{k+1} = k(f+h)+f \quad k=0,....,s$$
$$n = L_{s+1} = s(f+h)+f$$

On the other hand instalment amounts follow an arithmetic adjustment process:

$$d_{kj} = d + (j+k-1+Y_k)v = d + (j-1-kh)v \quad \begin{matrix} k=0,\dots,s \\ j=M_k,\dots,L_{k+1} \end{matrix} \qquad (6)$$

Where:

$$Y_k = \sum_{t=1}^{k} L_t - M_t = \sum_{t=1}^{k}\left[(t-1)(f+h)+f-t(f+h)-1\right] = -k(h+1),$$

Formula can be derived. Since present value of the loan are equal to present value of sum of all instalments,

$$\rho = \sum_{k=0}^{s} \sum_{j=M_k}^{L_{k+1}} d_{kj} R^{-j} = dA + vB \qquad (7)$$

is achieved.

Where:

$U_1 = \left(R^{-f} - 1\right)$, $U_2 = \left(R^{-(f+h)} - 1\right)$, $U_3 = \left(R^{-(f+h)(s+1)} - 1\right)$, $A = \dfrac{-U_1U_3}{rU_2}$,

$B = \dfrac{fU_1}{rU_2}\left[U_3\left(\dfrac{1}{U_2} - \dfrac{1}{fr} - 1 - s - \dfrac{1}{U_1}\right) - 1 - s\right]$ (see Appendices)

Considering equation (7), following formulas are achieved:

$$d = \frac{p - vB}{A} \qquad 8)$$

$$v = \frac{p - dA}{B} \qquad 9)$$

Equations (8) and (9) are general formulas or models derived under rhythmic skip assumption.

Numerical Example 1

An automobile has a price tag of 60000 dollars. Yet it has been purchased with monthly repayment loan. A loan schedule will be set up with 10 months periodic payments and then 2 months of skips for a total of 34 months. Periodic payment amounts will increase with 10 dollars from one instalment to another successively, excluding skip periods. We calculate the monthly instalment amounts setting up the monthly interest rate to %1.

Then problem inputs can be summarized as below:

p: 60000, *f*: 10, *h*: 2, *s*: 2, *n*: 34, *v*: 10, *r*: 0.01

Using equation (8), the first instalment amount is calculated as *d* = 2231.945. Later, other instalment amounts are calculated from equation (6).

Table 1
Loan Repayment Schedule for Numerical Example 1

Months	Instalment (periodic payment) amounts	Balance due (dollars)
0	-	60000
1	2231.945	58368.055
2	2241.945	56709.791
3	2251.945	55024.943
4	2261.945	53313.248
5	2271.945	51574.435
6	2281.945	49808.235
7	2291.945	48014.372
8	2301.945	46192.571
9	2311.945	44342.552
10	2321.945	42464.032
11	0	42888.672
12	0	43317.559
13	2331.945	41418.790
14	2341.945	39491.033
15	2351.945	37533.998
16	2361.945	35547.393
17	2371.945	33530.922
18	2381.945	31484.286
19	2391.945	29407.184
20	2401.945	27299.311
21	2411.945	25160.359
22	2421.945	22990.017
23	0	23219.918
24	0	23452.117
25	2431.945	21254.693
26	2441.945	19025.295
27	2451.945	16763.603
28	2461.945	14469.294
29	2471.945	12142.042
30	2481.945	9781.517
31	2491.945	7387.387
32	2501.945	4959.316
33	2511.945	2496.964
34	2521.945	0

3. A Model for Loan Repayment with Equal Periodic Payments and Rhythmic Skips

Taking v = 0, the model for loan repayment with periodic instalments that has arithmetic change and rhythmic skips will reduce to a model for loan repayment with equal periodic payments and rhythmic skips. Then general formulas of (7) and (8) will modify to (10) and (11):

$$p = \frac{d\left(1-R^{-f}\right)\left(R^{-(f+h)(s+1)}-1\right)}{r\left(R^{-(f+h)}-1\right)} \qquad 10)$$

$$d = \frac{rp\left(R^{-(f+h)}-1\right)}{\left(1-R^{-f}\right)\left(R^{-(f+h)(s+1)}-1\right)} \qquad 11)$$

Numerical Example 2

An automobile which has the advance price of 45000 dollars has been purchased with a loan schedule which includes consecutive 5 months periodic payments and a month of skip for a total of 23 months. We calculate the monthly periodic payment amounts setting up the monthly interest rate to %1.

The problem inputs are given in below:

p: 45000, *f*: 5, *h*: 1, *s*: 3, *n*: 23, *r*: 0.01

Using the equation (11), the monthly instalment amounts become *d* = 2529.467.

Table 2

Loan Repayment Schedule for Numerical Example 2

Months	Instalment (periodic payment) amounts	Balance due (dollars)
0	-	45000
1	2529.467	42920.533
2	2529.467	40820.271
3	2529.467	38699.007
4	2529.467	36556.530
5	2529.467	34392.628
6	0	34736.555

Months	Instalment (periodic payment) amounts	Balance due (dollars)
7	2529.467	32554.453
8	2529.467	30350.531
9	2529.467	28124.569
10	2529.467	25876.348
11	2529.467	23605.644
12	0	23841.701
13	2529.467	21550.651
14	2529.467	19236.690
15	2529.467	16899.590
16	2529.467	14539.119
17	2529.467	12155.043
18	0	12276.594
19	2529.467	9869.893
20	2529.467	7439.125
21	2529.467	4984.049
22	2529.467	2504.422
23	2529.467	0

4. Conclusion

Current economies are lay out with higher variability in the cases from globalism. So, the customer types and the debt volumes are also changed in globally markets. The customers or investors are demanded the new repayment financial models for their variable debts levels market conditions. As a result of these changes and innovations, the new repayment models must be derived in alternatively for loan payment model that most commonly used by banks or credit institutions.

The notion of loan repayments rest on the principle that present value of sum of the periodic payments are equal to present value of the loan total. Loan amortization models differ from each other in distribution of instalments. Equal, geometric gradient and arithmetic gradient periodic payments exist in conventional models. From a customer oriented financial institution's view point, it can be functional and can help creating satisfying customers to offer alternative loan repayment plans that fit individual's personal income flow, which may change along the year. Alternative loan repayment models with alternative periodic payment plans should facilitate reaching to additional customers, to those otherwise do not shop at the market. This idea led to derivation of the loan amortization models that have arbitrary skips by Formato (1992), Moon (1994), Eroğlu (2000), Eroglu and Karaoz (2002), Eroglu et al (2011), Eroglu and Ozdemir (2012), Eroglu et al. (2013), Eroğlu (2013a), Eroglu (2013b) and Eroglu et al. (2014), Aydemir and Eroglu (2014).

In this study, general formulas derived under two assumptions in this current paper. First, periodic payments adjust with arithmetic sequence. Second, periodic payments take breaks with rhythmic skip intervals along the loan term. Later, by setting the arithmetic change to zero, a general formula for loan repayment with equal periodic instalments that also has rhythmic skips has been derived. The results are obtained with the numerical examples with a repayment schedule is shown in clearly and understandable. The results of this study indicate that numerical examples have been used in order to show the validity of the models.

This study is important in terms of the determination of the amount of payment for customers and pays the appropriate balance. This study is also significant in order to increase repayment options in any debt payment model and to access more consumers when these models are applied by banks or credit institutions.

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APPENDIX

$$p = \sum_{k=0}^{s} \sum_{j=M_k}^{L_{k+1}} d_{kj} R^{-j} = \sum_{k=0}^{s} \sum_{j=k(f+h)+1}^{k(f+h)+f} [d + (j - 1 - kh)v] R^{-j}$$

$$= \left[\sum_{k=0}^{s} \sum_{j=k(f+h)+1}^{k(f+h)+f} dR^{-j} \right] + \left[\sum_{k=0}^{s} \sum_{j=k(f+h)+1}^{k(f+h)+f} (j - 1 - kh) v R^{-j} \right]$$

$$= d \left[\sum_{k=0}^{s} \sum_{j=k(f+h)+1}^{k(f+h)+f} R^{-j} \right]$$

$$+ v \left\{ \left[\sum_{k=0}^{s} \sum_{j=k(f+h)+1}^{k(f+h)+f} -(1 + kh) R^{-j} \right] + \left[\sum_{k=0}^{s} \sum_{j=k(f+h)+1}^{k(f+h)+f} j R^{-j} \right] \right\} = dA + v(B_1 + B_2)$$

$$= dA + vB$$

Where:

$$U_1 = (R^{-f} - 1)$$

$$U_2 = (R^{-(f+h)} - 1)$$

$$U_3 = (R^{-(f+h)(s+1)} - 1)$$

$$B_1 = \frac{U_1}{rU_2} \left\{ U_3 + h \left[s(U_3 + 1) - \frac{U_3}{U_2} + 1 \right] \right\}$$

$$B_2 = \frac{-(f + h)U_1}{rU_2} \left[s(U_3 + 1) - \frac{U_3}{U_2} + 1 \right] + \frac{U_1 U_3}{rU_2} \left[-1 - \frac{1}{r} - f\left(1 + \frac{1}{U_1}\right) \right]$$

$$A = \frac{-U_1 U_3}{rU_2}$$

$$B = B_1 + B_2 = \frac{fU_1}{rU_2} \left[U_3 \left(\frac{1}{U_2} - \frac{1}{fr} - 1 - s - \frac{1}{U_1} \right) - 1 - s \right]$$

$$B_1 = \sum_{k=0}^{s} \sum_{j=k(f+h)+1}^{k(f+h)+f} -(1+kh)R^{-j} = -\sum_{k=0}^{s} \sum_{j=k(f+h)+1}^{k(f+h)+f} (1+kh)R^{-k(f+h)-1}\left(\frac{R^{-f}-1}{R^{-f}-1}\right)$$

$$= \frac{\left(R^{-f}-1\right)}{r}\left(\sum_{k=0}^{s}(1+kh)R^{-k(f+h)}\right)$$

$$B_1 = \frac{\left(R^{-f}-1\right)}{r}\left[\left(\sum_{k=0}^{s} R^{-k(f+h)}\right) + \left(h\sum_{k=0}^{s} kR^{-k(f+h)}\right)\right]$$

$$= \frac{\left(R^{-f}-1\right)}{r}\left[\left(\sum_{k=0}^{s} R^{-k(f+h)}\right) + \left(h\sum_{k=1}^{s}\sum_{t=k}^{s} R^{-t(f+h)}\right)\right]$$

$$= \frac{\left(R^{-f}-1\right)}{r}\left[\left(\frac{R^{-(f+h)(s+1)}-1}{R^{-(f+h)}-1}\right)\right.$$

$$\left. + \left(\frac{h}{R^{-(f+h)}-1}\left[sR^{-(f+h)(s+1)} - \left(\frac{R^{-(f+h)(s+1)}-R^{-(f+h)}}{R^{-(f+h)}-1}\right)\right]\right)\right]$$

$$= \frac{R^{-f}-1}{r(R^{-(f+h)}-1)}\left\{R^{-(f+h)(s+1)}-1\right.$$

$$\left. + h\left[sR^{-(f+h(s+1))} - \left(\frac{R^{-(f+h)(s+1)}-R^{-(f+h)}}{R^{-(f+h)}-1}\right)\right]\right\}$$

$$= \frac{U_1}{rU_2}\left\{U_3 + h\left[s(U_3+1) - \frac{(U_3+1)-(U_2+1)}{U_2}\right]\right\}$$

$$= \frac{U_1}{rU_2}\left\{U_3 + h\left[s(U_3+1) - \frac{U_3}{U_2} + 1\right]\right\}$$

MODELING ASYMMETRIC VOLATILITY IN THE CHICAGO BOARD OPTIONS EXCHANGE VOLATILITY INDEX

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Abstract

Empirical studies have shown that a large number of financial asset returns exhibit fat tails (leptokurtosis) and are often characterized by volatility clustering and asymmetry. This paper considers the ability of the asymmetric GARCH-type models (TGARCH, EGARCH, APGARCH) to capture the stylized features of volatility in the Chicago Board Options Exchange Volatility Index (VIX). We analyzed daily VIX returns for the period September 26th, 2012 - September 27th, 2017. The results of this paper suggest that in the presence of asymmetric responses to innovations in the market, the EGARCH (1,1) Student-t model which accommodates the kurtosis of VIX return series is preferred.

Keywords: asymmetry, volatility, response to market innovation

JEL Classification: C22, C58, G15

1. Introduction

VIX is the ticker symbol for the Chicago Board Options Exchange[1] (CBOE) Volatility Index, which represents market expectations of volatility over the next 30 days (CBOE, 2017a). VIX indices are computed for various instruments. The most important VIX index is the S&P 500 VIX index, which is computed using data

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[1] *The first exchange to list standardized, exchange-traded stock options began its first day of trading on April 26, 1973, in a celebration of the 125th birthday of the Chicago Board of Trade (CBOT).*

from S&P 500 options contracts. Option contract prices depend on many factors, the most important of which are the strike price, the price of the underlying instrument, the time to maturity and the expected future price volatility of the underlying instrument. When expected volatility is high, option prices are high. Carefully chosen averages of option prices thus can estimate volatility. VIX options give traders a way to trade volatility without having to factor in the price changes of the underlying instrument, dividends, interest rates or time to expiration - factors that affect volatility trades using regular equity or index options. VIX options allow traders to focus almost exclusively on trading volatility (Ahoniemi, 2006: 2-3).

The VIX uses prices of various S&P 500 options with expirations between 23 and 37 days to measure traders' expectations of volatility. The VIX helps us measure sentiment by telling us how much traders are willing to pay for these options. Typically, the VIX rises when traders are worried about downside risk. Because when traders are worried about downside risk, they'll pay higher prices for downside protection through options. This illustrates how the VIX rises when traders are scared and markets are coming under pressure. Again, because traders were willing to pay up big for downside protection through S&P 500 options (CBOE, 2017b).

The VIX was the first successful attempt at implementing a volatility index. When the index was first conceived in 1993, the methodology was based on a Black-Scholes pricing model given a known market option price (a weighted measure of the implied volatility of eight S&P 100 at-the-money put and call options). The method of calculation for the VIX has varied through time. In 2004, the VIX expanded to use options based on a broader index, the S&P 500, which allows for a more accurate view of investors' expectations on future market volatility (Hancock, 2012: 284-285).

Whaley (2000) points out on the CBOE's 'investor fear gauge' index; it is the forward-looking measure of future stock market volatility, and this index is constructed by market participants through observed option prices. The highest level of VIX implies greater investor's fear. Whaley argues that VIX is more a barometer of invertors' fear (investor sentiment) of the downside risk. Higher VIX levels indicate that the market's expectation of 30-day forward volatility is increased.

One advantage of the VIX is its negative correlation with the movements in the market (S&P 500). According to the CBOE's own website, since 1990 the VIX has moved opposite the S&P 500 Index (SPX) 88% of the time. The inverse relationship between market volatility and stock market returns suggest a diversification benefit which can significantly reduce portfolio risk (Dennis et al., 2006: 382-383; Brandt and Kang, 2004). The national and international economic, political and/or social problems (shocks) affect especially the financial markets with high liquidity and increase the volatility of these markets. Movements of the VIX are largely dependent on market reactions. This means global investors saw uncertainty in the market and decided to take profits/gains or realize/stop losses.

The purpose of this study is to examine the comparative performance of asymmetric volatility models (TGARCH, EGARCH and APGARCH) under Student-*t* and GED distributions by using daily returns of CBOE Volatility Index (VIX). The remainder of this paper proceeds as follows. The section 2 details the Asymmetric GARCH-type models (TGARCH, EGARCH, APGARCH) methodology. The section 3 describes the VIX-CBOE Volatility Index returns data to be used in this study and presents the empirical results. The robustness of these findings is assessed using the Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC), Hannan-Quinn Criterion (HQC), log-likelihood (LL) values. The section 4 contains some concluding remarks.

2. Methodology

Empirical studies have shown that a large number of financial asset returns exhibit fat tails (leptokurtosis) and are often characterized by volatility clustering and asymmetry in volatility. Asset returns are approximately uncorrelated but not independent through time as large (small) price changes tend to follow large (small) price changes. This temporal concentration of volatility is commonly referred to as volatility clustering and it was not fully exploited for modeling purposes until the introduction of the ARCH model by Engle (1982) and Generalized ARCH (GARCH) model by Bollerslev (1986).

Both the ARCH and GARCH models allow taking the first two characteristics into account, but their distributions are symmetric and therefore fail to model the third stylized fact, namely the “leverage effect” (see Black 1976, Christie 1982 and Nelson 1991). Almost all financial returns data commonly exhibits an asymmetry in that positive and negative shocks to the market do not bring forth equal responses. The underlying concept is that negative shocks increase conditional volatility more than positive shocks, hence there is asymmetry on the impact of good and bad news on the riskiness of the stock market.

Due to an increasing number of empirical evidences saying that negative (positive) returns are generally associated with upward (downward) revisions of conditional volatility, this phenomenon is often referred to as asymmetric volatility in the literature (Goudarzi, 2011). To solve this problem, many nonlinear extensions of the GARCH model have been proposed. Among the most widely spread asymmetric volatility models are the GJRGARCH (Glosten, Jagannathan and Runkle GARCH)) or TGARCH (Threshold-GARCH), EGARCH (Exponential GARCH) and APGARCH (Asymmetric Power GARCH) models. Here, the basic definitions and theoretic properties of the models are discussed.

***TGARCH Model*:** In order to verify the existence of asymmetric volatility in VIX returns, one of the model were introduced independently by Zakoian (1994) and Glosten, Jaganathan, and Runkle (1993). By assigning a dummy variable to negative returns, they were able to allow asymmetric effects of good and bad news on conditional volatility. It is also known as Threshold GARCH (TGARCH) model since we consider $\varepsilon_{t-1} = 0$ as a point of separation of the impacts of negative and positive shocks (Enders, 2004). The generalized specification for the conditional variance is given by:

$$\sigma_t^2 = \omega_0 + \sum_{i=1}^{p}\left(\alpha_i + \gamma_i N_{t-i}\right)\varepsilon_{t-i}^2 + \sum_{j=1}^{q}\beta_j\sigma_{t-j}^2 \quad \left(\omega_0 > 0 \text{ and } \alpha_i, \beta_j, \gamma_i \geq 0\right)$$

$$\left.\begin{array}{l} \varepsilon_{t-i} < 0 \text{ ise, } 1 \ (\textit{bad news}) \\ \varepsilon_{t-i} \geq 0 \text{ ise, } 0 \ (\textit{good news}) \end{array}\right\} = N_{t-i}$$

where σ_t^2 is the conditional variance at time t, α_i is the coefficient for the ARCH process, N_{t-i} is asymmetric effects of good and bad news on conditional volatility and β is the coefficient for the GARCH process. In addition if $\gamma_i \neq 0$ news impact is asymmetric and $\gamma_i > 0$ leverage effect exists (Brooks, 2008: 406).

EGARCH Model: The EGARCH or Exponential GARCH model was proposed by Nelson (1991). The EGARCH model is given by:

$$\ln\left(\sigma_t^2\right) = \omega_0 + \sum_{i=1}^{p} \alpha_i \frac{\left|\varepsilon_{t-i}\right| + \gamma_i \varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

Note that the left-hand side is the log of the conditional variance. This implies that the leverage effect is exponential, rather than quadratic, and that forecasts of the conditional variance are guaranteed to be non-negative. The presence of leverage effects can be tested by the hypothesis that. In the equation γ_i represent leverage effects which accounts for the asymmetry of the model. If $\gamma_i < 0$ it indicates leverage effect exist and if $\gamma_i \neq 0$ impact is asymmetric. The meaning of leverage effect bad news increase volatility.

APGARCH Model: The Generalized Asymmetric Power ARCH (APGARCH) model, which was introduced by Ding, Granger and Engle (1993), is presented in the following framework:

$$\sigma_t^{\delta} = \omega_0 + \sum_{i=1}^{p} \alpha_i (\left|\varepsilon_{t-i}\right| - \gamma_i \varepsilon_{t-i})^{\delta} + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^{\delta}$$

$$\left(\omega_0 > 0,\ \alpha_i \geq 0,\ \beta_j \geq 0,\ \delta \geq 0 \text{ and } \left|\gamma_i\right| \leq 1\right)$$

where ω_0 is a constant parameter, α_i and β_j are the standard ARCH and GARCH parameters, γ_i is the leverage parameter and δ is the parameter for the power term. A positive (resp. negative) value of the γ_i means that past negative (resp. positive) shocks have a deeper impact on current conditional volatility than past positive (resp. negative) shocks. In the APGARCH model, the power parameter δ of the standard deviation can be estimated rather than imposed, and the optional γ_i parameters are added to capture asymmetry.

The model imposes a Box and Cox (1964) transformation in the conditional standard deviation process and the asymmetric absolute innovations. In the APGARCH model, good news ($\varepsilon_{t-i} > 0$) and bad news ($\varepsilon_{t-i} < 0$) have different predictability for future volatility, because the conditional variance depends not only on the magnitude but also on the sign of ε_t.

Failure to capture fat-tails property of high-frequency financial time series has led to the use of non-normal distributions to better model excessive third and fourth moments. To accommodate this, rather than to use Normal (Gaussian) distribution the Student-*t* distribution and Generalized Error Distribution (GED) used to employ GARCH-type models (Mittnik et al. 2002: 98). Bollerslev (1987) tried to capture the high degree of leptokurtosis that is presented in high frequency data and proposed the Student-*t* distribution in order to produce an unconditional distribution with fat tails.

3. Data and Empirical Results

The section shows the empirical results of models. The VIX returns are analyzed. The characteristics of the data are presented in the first subsection. The second subsection shows the estimated results of asymmetric GARCH-type model specifications and the corresponding qualification tests.

3.1. Data

In this study, we used daily VIX returns for the period September 26th, 2012 – September 27th, 2017. The VIX returns are calculated by log return $r_t = \ln(p_t / p_{t-1})$ of the closing values. The data used in the study is obtained from the Yahoo Finance. Table 1 presents the descriptive statistics for VIX return series (RVIX).

Table 1

Descriptive statistics

	RVIX
Mean	-0.000475
Minimum	-0.299831
Maximum	0.401011
Standard Deviation	0.075429
Skewness	0.745042
Excess Kurtosis	4.026095
Jarque-Bera (p-value)	965.26 (0.000)
ADF-Test (N, 0)*	-37.49039
PP-Test (N, 0)*	-45.27840
ARCH-LM (p-value)	54.01 (0.000)

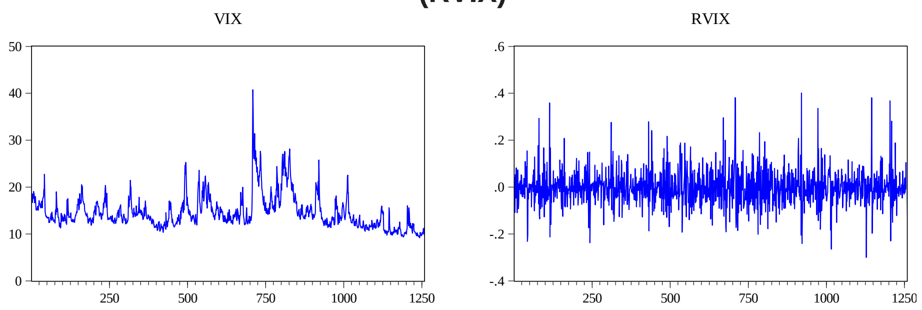
*Notes: * (N, 0) indicates that there is no constant and no trend in the regression model with lag=0.*

According to descriptive statistics, volatility, as measured by standard deviation is high (0.0745042). It is not surprising that this series exhibit asymmetric and leptokurtic (fat tails) properties. The VIX return series have positive skewness, and the excess kurtosis exceeds zero indicating fat tails and leptokurtic distribution. Thus, the VIX returns are not normally distributed. Additionally, by Jarque-Bera statistic and corresponding *p*-value, we reject the null hypothesis that returns are well approximated by the normal distribution. For this reason, in this study we used the Student-*t* distribution and GED distribution, which takes into account fat tail problem. ARCH-LM statistics highlight the existence of conditional heteroskedastic ARCH effect. The VIX return series are subjected to two unit root tests to determine whether stationary I(0). The Augmented-Dickey–Fuller (ADF) and Phillips–Peron (PP) test statistics reject the hypothesis of a unit root at the 1% level of confidence. MacKinnon critical value at the 1% confidence level is -2.57.

As well as descriptive statistics, examining the VIX closing value and return series (RVIX) graphs in Figure 1 shows the volatility clustering in several periods. Volatility clustering which means that there are periods of large absolute changes tend to cluster together followed by periods of relatively small absolute changes.

Figure 1

Daily CBOE Volatility Index Series (VIX) and Log-Return Series (RVIX)



3.2. Estimation Results

In this subsection, the TGARCH, EGARCH and APGARCH models are estimated for VIX return series under Student-*t* and GED distributions. The standard of model selection is based on in-sample diagnosis including Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC), Hannan-Quinn criterion (HQC), log-likelihood (LL) values, and Ljung-Box Q and Q^2 statistics on standardized and squared standardized residuals respectively. Under every distribution, the model which has the lowest AIC and SIC or highest LL values and passes the Q-test simultaneously is adopted.

Table 2 presents the results of this estimation procedure and from this table one can see that all of the ARCH and GARCH coefficients are statistically significant at the 1% confidence level. Further, β is close to 1 but significantly different from 1 for all models, which indicates a high degree of volatility persistence. β values suggesting that there are substantial memory effects. Furthermore, all models are stationary in the sense that stationary coefficients[2] are lower than 1.

Table 2

Asymmetric GARCH-Type Model Estimation Results

	TGARCH (1,1)	EGARCH (1,1)	APGARCH (1,1)
μ	-0.001529 [-0.9484[b]]	0.000168 [0.1010[b]]	-0.000301 [-0.1870[b]]
ω	0.000757 [4.59691]	-0.498651 [-4.60646]	0.009957 [2.36946[a]]
α	0.335758 [4.75128]	0.072726 [1.96258[a]]	0.141330 [7.49857]
β	0.718076 [16.2605]	0.918876 [50.8638]	0.821489 [25.9540]
γ	-0.373928 [-5.14374]	0.292190 [8.69130]	-0.999999 [-4.80000]
δ	-	-	0.833803 [6.25461]

[2] *For TGARCH model* $(\alpha+\beta+k\gamma)<1$, *for EGARCH model* $\beta<1$ *and for APGARCH model* $\alpha_i E(|z|-\gamma_i z)^\delta+\beta_j<1$.

	TGARCH (1,1)	EGARCH (1,1)	APGARCH (1,1)
LL	1,628.06	**1,637.12**	1,634.58
AIC	-2.5808	**-2.5953**	-2.5896
SIC	-2.5563	**-2.5707**	-2.5610
HQC	-2.5716	**-2.5860**	-2.5789
Q(10)	24.477 (0.006)	26.408 (0.003)	26.320 (0.003)
Q^2(10)	4.8311 (0.902)	3.9096 (0.951)	3.9334 (0.950)
ARCH-LM	0.503458 (0.4780)	0.106238 (0.7445)	0.041699 (0.8382)

a denotes 5% significance level, b denotes not significant; z-statistics of corresponding tests in brackets. LL is the value of the maximized log-likelihood, AIC-Akaike Information Criterion, SIC-Schwarz Information Criterion and Hannan-Quinn criterion (HQC). Q(10) and Q^2(10) are the Ljung-Box statistics for remaining serial correlation in the standardized and squared standardized residuals respectively using 10 lags with p-values in parenthesis. ARCH-LM denotes the ARCH test statistic with lag 1.

The asymmetric volatility models include a leverage term (γ) which allows positive and negative shocks of equal magnitude to elicit an unequal response from the market. Table 3 presents details of this leverage term and reveals that for all models fitted; the estimated coefficient was negative (for EGARCH positive but according to the EGARCH model, the coefficient is interpreted in opposite direction) and statistically significant. This means that past positive shocks lead to higher subsequent volatility than past negative shocks (asymmetry in the conditional variance).

From Table 2, the evidence of long memory process could be also found in the results of the model estimation because the power term (δ) of APGARCH model is 0.833803. The estimated power term was significantly different from two. This means that, the optimal power term was some value other than unity or two which would seem to support the use of a model which allows the power term to be estimated.

The results given in Table 2 show that the all models succeed in taking into account all the dynamical structure exhibited by the returns and volatility of the returns as the Ljung-Box statistics for up to

10 lags on the standardized residuals (Q) significant at the 5% level and the squared standardized residuals (Q^2) non-significant at the 5% level for VIX return series. Also, there is no evidence of remaining ARCH effects according to the ARCH-LM test statistic with lag 1.

In summary, ranking by AIC, SIC, HQC and LL favors the EGARCH (1,1) Student-*t* specification in VIX return series. To conserve space, the results of the models with other distributions declined to present, but they are available upon request.

4. Conclusion

The VIX is based on S&P 500 data. The VIX can be used as a predictor for S&P 500 returns, stock market volatility, economic activity, financial instability, financial crises etc. Empirical studies have shown that a large number of financial asset returns exhibit fat tails (leptokurtosis) and are often characterized by volatility clustering and asymmetry. The long-memory properties of this index have been investigated in numerous empirical studies that have provided mixed results.

The purpose of this study is to examine the comparative performance of asymmetric volatility models (TGARCH, EGARCH and APGARCH) under Student-*t* and GED distributions by using daily returns of CBOE Volatility Index (VIX). The results of models highlight that in the presence of asymmetric responses to innovations in the market, the EGARCH (1,1) Student-*t* model which accommodates the kurtosis of VIX return series is preferred. The estimation results indicate that strong leverage effects are present in VIX returns. Further, in VIX return series the volatility persistence is higher. Thus, shocks in the VIX return series have substantial memory effects.

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ANALYSIS OF HOW THE EUROPEAN STOCK MARKETS PERCEIVE THE DYNAMICS OF MACROECONOMIC INDICATORS THROUGH THE SENTIMENT INDEX AND THE PURCHASING MANAGERS' INDEX

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Abstract

In this article, we intend to analyse how the European stock markets perceive the dynamics of macroeconomic indicators in terms of the sentiment index and the purchasing managers' index. For this research, we focused on the countries of the European Union and applied an econometric event study, which consisted in the analysis of the evolution of the logarithmic returns of the stock indices for 27 countries of the European Union and for the euro area for the period January 2007 - November 2017. The results showed immediate reactions with a higher intensity in March 2015 for the SentiMent index and for March 2016 for the PMI. The frequency and amplitude of reactions are different from country to country; often, a high frequency of reactions in one country is not reflected in a very high amplitude response.

Keywords: capital markets, sentiment indices

JEL Classification: G19, G32

1. Short introduction

The specific features of stock markets provide us with very high frequency statistics, which allows the use of empirical methods to analyse the immediate impact of certain events on the evolution of stock indices.

For the purpose of this research, we have chosen the countries of the European Union as an area of interest, from an economical and financial point of view, the Romanian economy being

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linked to this area. The use of survey-based indices to capture how the economy operates as a whole or in certain sectors is more and more common in both practice and academia.

The two indices chosen to assess the impact of changes on stock markets have a wide coverage, both in terms of countries for which they are calculated and their use in practice and research, the indices being published monthly and intensely pursued by the economic press.

The methodology of calculating the indices differs, starting from the basic approach - "opinions" versus "facts", the choice of two variants of indicators for research being motivated by the desire to capture a reaction of the stock markets to indicators that represent in different ways the macroeconomic reality.

The advantage of choosing such indicators at the expense of classical macroeconomic indicators is access to calculated and published monthly indicators compared to the trimestral publication of some macroeconomic indicators, and thus a much higher frequency of the data series.

Another aspect worth mentioning is that once published, statistical data is no longer reviewed, as is the case of many macroeconomic indicators.

Baker and Wurgler (2007) published an investigation of sentiment indicators on stock markets, while Rakovska and Svoboda (2016) made an analysis of their application in financial research. Sibley et al. (2016) explore the information content of this kind of indexes. Bormann (2013) presents an interesting explanation on the sentiment of the indices and sentiment concept.

In previous research, using models from the GARCH family and a MIDAS methodology, Lupu et al. (2016) explored the linkage between sentiment indices and the volatility of stock market indices, concluding that the risk associated with benchmark indices is higher than those specific to sustainability related counterparts.

2. Description of indices

2.1. Sentiment Index (SentiMent)

The European Commission has been calculating the *sentiment index* since 1985. Surveys conducted by the European Commission provide monthly judgments and forecasts on various aspects of economic business in distinct sectors of the economy:

industry, services, construction, retail and consumers. On the basis of the obtained results, the Commission computes and publishes a composite index monthly (the last working day of each month for the current month) to reflect overall perceptions and industry expectations in a one-dimensional index for member countries and candidate countries (European Commission, 2017).

For calculating the composite index (used in this study), the above-mentioned sectors as components of this index are assigned some weightings:

- Industry sector: 40%;
- Services sector: 30%;
- Consumers sector: 20%;
- Construction sector: 5%;
- Retail sector: 5%.

Assigned weights were determined using two criteria: sector representativeness and performance tracking against the reference variable. Considering the composite index, the reference variable is GDP growth, which represents the change in the economy as a ensemble, used to test the performance of the composite index.

This indicator summarizes optimistic or pessimistic expectations regarding the economic developments, being very useful in monitoring and forecasting the business cycle. An index value above 100 represents a value of the economic sentiment above average, and according to the configuration, in 68% of the cases, the sentiment index will be between 90 and 110. The usefulness of the index is related to economic surveillance, the realization of short-term forecasts and in economic research.

2.2. Purchasing managers index (PMI)

Markit Economics develops the Purchasing Managers Index, PMI) based on monthly questionnaires addressed to companies. This index gives an overview of what is happening in the private environment of the economy by tracking variables such as production, new orders, stock levels, employment rates and prices in various sectors of the economy (industry, construction, trade and services).

According to the calculation methodology, PMI is calculated and published monthly, is based rather on facts than on opinions, and uses the same method in all countries, thus providing a comparable basis for assessing the production sector.

The index is widespread, with a large coverage in the press, and is used by corporate managers, economic analysts in financial institutions and central banks, the embedded information being useful for building monetary policy decisions.

3. Description of data and methodology

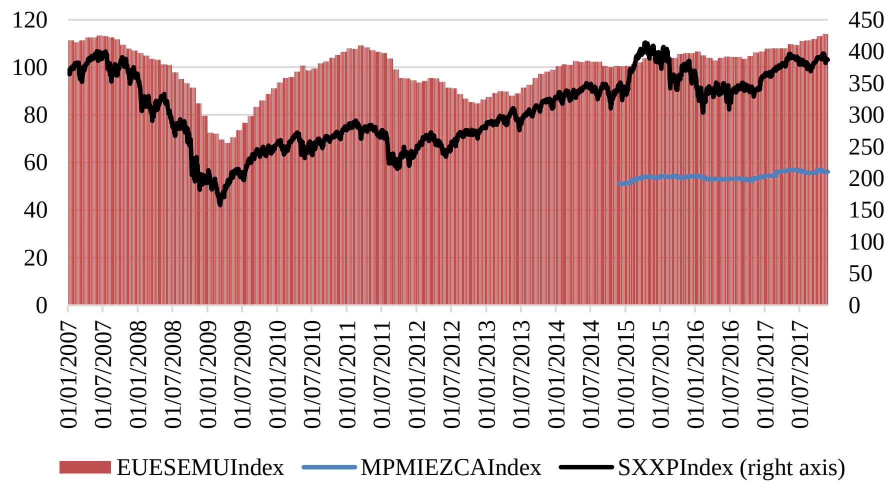
The econometric event study consisted in the analysis of the evolution of the logarithmic returns of the stock indices for 27 countries of the European Union and for the euro area during January 2007 and November 2017.

The main purpose of this analysis was to capture the reaction of these returns to changes in the PMI published by Markit Economics, and to changes in SentiMent index values, calculated by the European Commission and that incorporates analysts' views regarding the economic policies in the countries of the European Union. For countries for which PMI is not calculated, the PMI calculated for the euro area was used.

An example of the evolution of these three indicators (SentiMent, PMI and stock index) for the euro area and Europe is shown in Figure 1.

Figure 1

Evolution of the SentiMent index, the PMI for the euro area and the STOXX 600 stock index for Europe (Jan. 2007 – Nov. 2017)



Source: Authors' processing using Bloomberg data

A first step of the analysis was to identify the moments in which the two indices (PMI and SentiMent) changed for each country in the sample. As a result of this research, different time points resulted for the analyzed countries, the frequency of searches for changes in index values being daily.

Therefore, the "event" was the change in the two categories of indices for each country. Correspondence of stock indices for each country with PMI and SentiMent indices is presented in the Table 1.

Table 1

Correspondence between stock indices, SentiMent indices and PMI for each country/area considered for analysis

Country	Stock Market Index	SentiMent indices	PMI
Austria	ATXIndex	EUESATIndex	MPMIEZCAIndex
Belgium	BEL20Index	EUESBEIndex	MPMIEZCAIndex
Bulgaria	SOFIXIndex	EUESBGIndex	MPMIEZCAIndex
Croatia	CROIndex	EUESHRIndex	MPMIEZCAIndex
Cyprus	CYSMMAPAIndex	EUESCYIndex	MPMIEZCAIndex
Czech Republic	PXIndex	EUESCZIndex	MPMIEZCAIndex
Denmark	KAXIndex	EUESDKIndex	MPMIEZCAIndex
Estonia	TALSEIndex	EUESEEIndex	MPMIEZCAIndex
Finland	EUESFIIndex	HEXIndex	MPMIEZCAIndex
France	CACIndex	EUESFRIndex	MPMIFRCAIndex
Germany	DAXIndex	EUESDEIndex	MPMIDECAIndex
Greece	ASEIndex	EUESGRIndex	MPMIGRMAIndex
Ireland	ISEQIndex	EUESIEIndex	MPMIEZCAIndex
Italy	FTSEMIBIndex	EUESITIndex	MPMIITMAIndex
Latvia	RIGSEIndex	EUESLVIndex	MPMIEZCAIndex
Lithuania	VILSEIndex	EUESLTIndex	MPMIEZCAIndex
Malta	MALTEXIndex	EUESMTIndex	MPMIEZCAIndex
UK	UKXIndex	EUESUKIndex	MPMIEZCAIndex
Netherlands	AEXIndex	EUESNLIndex	MPMINLMAIndex
Poland	WIGIndex	EUESPLIndex	MPMIEZCAIndex
Portugal	BVLXIndex	EUESPTIndex	MPMIEZCAIndex
Romania	BET_XTIndex	EUESROIndex	MPMIEZCAIndex
Slovakia	DWSKIndex	EUESSKIndex	MPMIEZCAIndex

Country	Stock Market Index	SentiMent indices	PMI
Slovenia	SBITOPIndex	EUESSIIndex	MPMIEZCAIndex
Spain	IBEXIndex	EUESESIndex	MPMIEZCAIndex
Sweden	OMXIndex	EUESSEIndex	MPMIEZCAIndex
Hungary	BUXIndex	EUESHUIndex	MPMIEZCAIndex
The euro area	SXXPIndex	EUESEMUIndex	MPMIEZCAIndex

Source: Authors' processing using Bloomberg, European Commission and Markit Economics data

For the event study (the event study methodology is presented in Lupu and Dumitrescu, 2010), the following analysis was performed for each change of the two categories of indices:

a) A sample of 700 transaction days was selected before the "event" date. For this data a simple GARCH (1,1) model was calibrated for the logarithmic returns of the stock index.

b) For each of the 10 days before the "event" the quadratic yields and the differences between them and the variance values estimated with the help of the GARCH model (1,1) were calculated. Quadratic yield reflects the variance value of that day. The difference between this and the model's estimated variance is the extent to which the model manages to explain the true stock market values in the immediate vicinity of the "event."

c) An average of these 10 differences has been calculated. This average is the extent to which the GARCH model (1.1) manages to explain on average the series of variants made during the 10 days before the "event".

d) For the "event" day and for each of the next 5 days, the GARCH (1.1) previously calibrated model was used to make variance forecasts. Thus, 6 variances were obtained, corresponding to each of the 6 days (the "event" day to which the next 5 days are added).

e) For these 6 days the quadratic returns were also used as measures of the actual variances that occurred during the period that followed the "event".

f) There were calculated 6 differences between quadratic yields and variances predicted by GARCH (1.1) and an analysis was made of the extent to which each of these differences was greater than 2 times the average difference for the period before the "event", respectively the difference calculated under (c).

g) For each two-fold over-lapse of this average difference, a significant financial market reaction to the PMI or SentiMent indexes was considered.

In order to synthesize the results of the analysis, we have calculated all the situations in which significant financial market reactions were recorded for each of the days when the two indices (PMI and SentiMent) have changed. At the same time, we calculated the averages for each of market reaction situations. These averages reflect the overall amplitude of market changes because of a particular event.

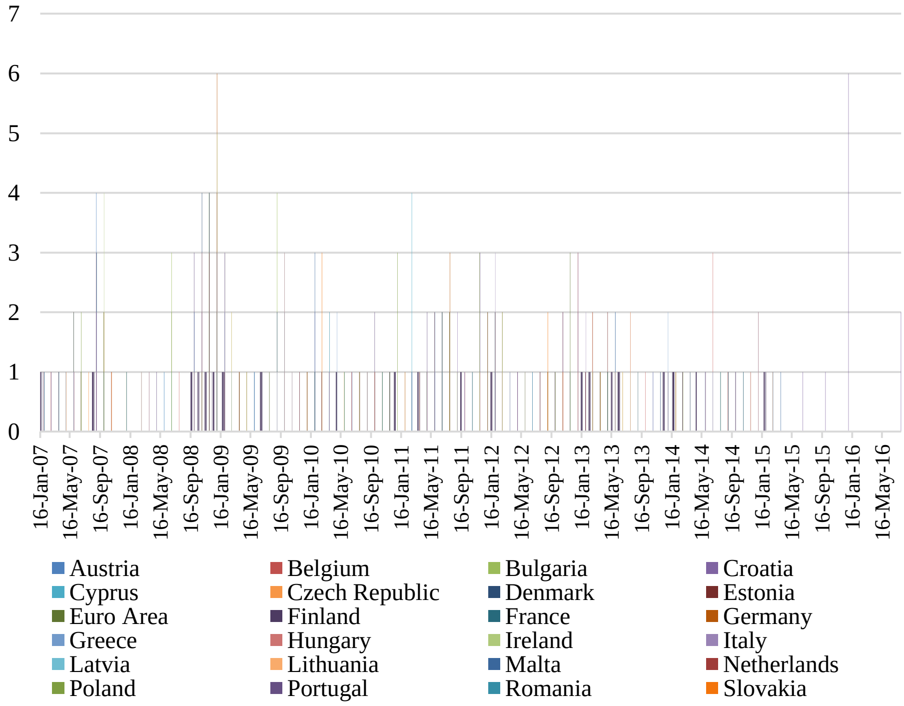
4. Obtained results

4.1. Results for the Sentiment Index (SentiMent)

The following graph (Figure 2) is more difficult to follow being populated with a lot of data, but it is useful for an overview, from which it can be deduced the general impact of the SentiMent index changes on the stock markets, namely the frequency and duration of significant changes (the number of days). It can be noticed that for all six days following the event significant changes took place in Germany and Croatia for changes in the SentiMent index of 31 December 2008 and 31 December 2015 respectively. Significant changes for longer periods (4 days) were in Austria, Sweden, Denmark, UK, Bulgaria and Cyprus. The impact of the SentiMent index changes on stock markets was more intense by the beginning of 2015, after which the frequency decreased.

Figure 2

The Reaction of European stock markets to changes in the European Commission's Sentiment Index (Jan. 2007 – July 2016)



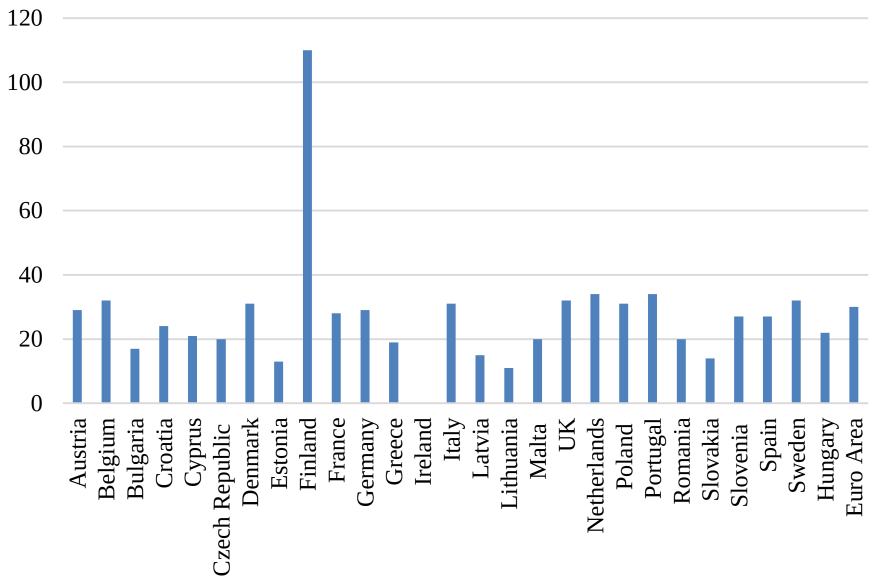
Source: Authors' processing using Bloomberg and European Commission data

Depending on the total number of days for which European stock markets responded significantly to changes in the SentiMent index (Figure 3), we note that Finland is ranked first with 110 days in total for the period under review (all stock market reactions are short-lived, all of them for one day), followed by the Netherlands with 34 days in total, Portugal with 34 days, Belgium, UK and Sweden with 32 days, Denmark, Italy and Poland with 31 days and the euro zone with 30 days.

For Romania, the total number of days in which reactions were recorded is 20, the longest significant response being recorded for three days at a change in the SentiMent index of August 31, 2017.

Figure 3

Total number of days for which European stock markets responded significantly to changes in the SentiMent index for each analysed country (January 2007 - July 2016)



Source: Authors' processing using Bloomberg and European Commission data

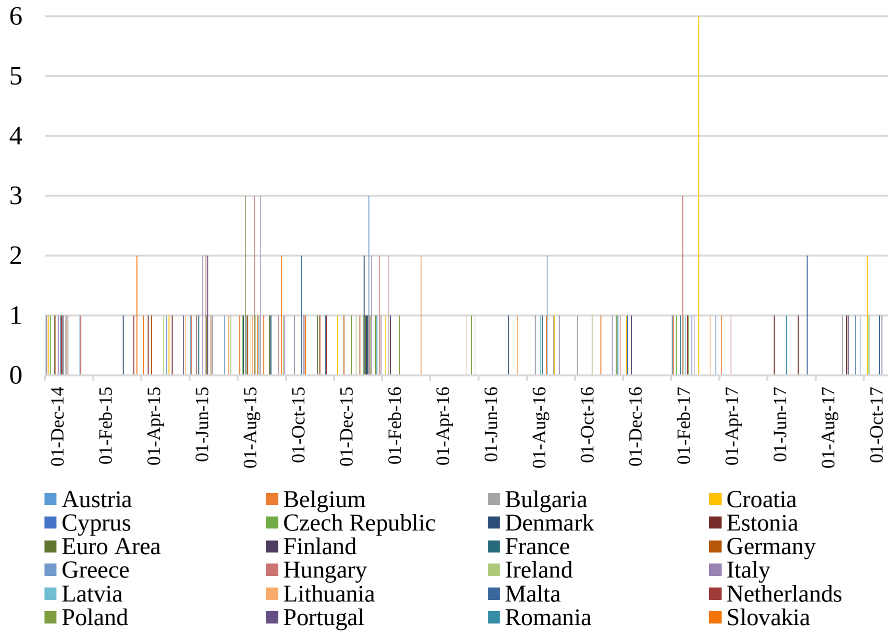
The magnitude of these changes (measured by multiple from the standard deviation resulting from the model calibration) is presented for each country in the graphs in Annex 1. The largest changes in the amplitude were recorded in Cyprus (164.00), Latvia (34.16), Lithuania (30.69), Greece (29.49), Croatia (29.06), Finland (22.46), Spain (18.92), the Czech Republic (17.57), Portugal (15.12), Poland (4.99), Slovakia (14.96), Romania (14.23). Ireland is the only country where there has been no change during the period under review.

4.2. Results for the Purchasing Managers' Index (PMI)

As in the previous case, although the next chart (Figure 4) is more difficult to follow, it can outline the overall impact of changes to the PMI on stock markets, namely the frequency and duration of significant changes (the number of days). It can be noticed that the only significant change for all six days pursued took place in Croatia for a change in PMI on 31 March 2017. Significant changes for longer periods were in the euro area, the Netherlands, the UK, Greece and Hungary (for three days) and in Slovakia (twice), Italy (twice), Portugal, Malta (twice), Denmark, Spain, Estonia, Lithuania, Slovenia and Croatia. The impact of PMI changes on stock markets was more intense until April 2016, after which the frequency decreased.

Figure 4

The Reaction of European stock markets to PMI changes (November 2014 - November 2017)



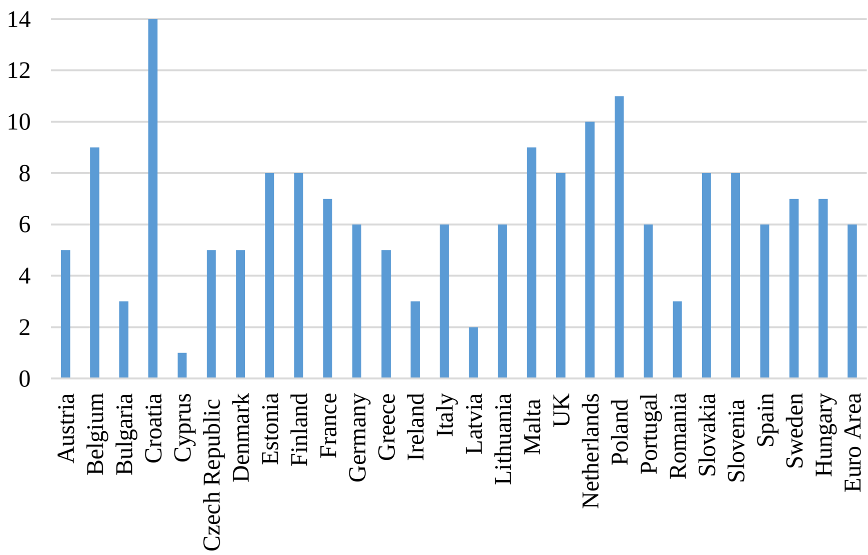
Source: Authors' processing using Bloomberg and Markit Economics data

Depending on the total number of days for which the European stock markets reacted significantly to PMI changes (Figure 5), we notice that the first position is Croatia with 14 days in total for the analysed period (one of the reactions had a duration of six days), followed by Poland with 11 days, the Netherlands with 10 days, Belgium and Malta with 9 days.

It should be noted that all countries have registered at least one significant reaction (Cyprus is the one-country country for a single day). This impact is also important given that only the Netherlands has a PMI calculated specifically for this country, with the other responding to the change in PMI for the euro area, with no specific PMI.

Figure 5

Total number of days for which European stock markets reacted significantly to PMI changes for each analysed country (November 2014 - November 2017)



Source: Authors' processing using Bloomberg and Markit Economics data

The magnitude of these changes (measured by multiple from the standard deviation resulting from the model calibration) is presented for each country in the graphs, in Annex 2. Although Croatia has a long-term response, its amplitude is reduced, with the standard deviation of only 5.99 higher. The biggest changes in the amplitude were recorded in Latvia (29.65), Greece (29.50), Bulgaria (16.47), Ireland (15.97), Finland (15.36), Slovakia (14,17), Romania (11,22, seventh place), Denmark (10,16), Hungary (10,06), Cyprus (8,36).

5. Final considerations

The global financial crisis has re-launched debates on the role of financial markets in spreading macroeconomic fluctuations. In this study, we attempted to identify the European stock markets' reactions to changes in sentiment indices and purchasing managers' indices, these indices being a mean of measuring macroeconomic status and evolution, different from the classic macroeconomic approach. The results showed immediate reactions, with a higher intensity by March 2015 for the SentiMent index and by March 2016 for PMI. Frequency and amplitude of reactions vary from country to country; often the high frequency of reactions in one country is not accompanied by a very high amplitude response.

For future research, a more complex, network-wide example, including multiple elements and their connections, could lead to early identification of vulnerabilities and the implementation of preventive measures. An interesting research would be to restore the analysis for various sectors of activity, but also to correlate with the economic and/or financial cycle.

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Electronic resources

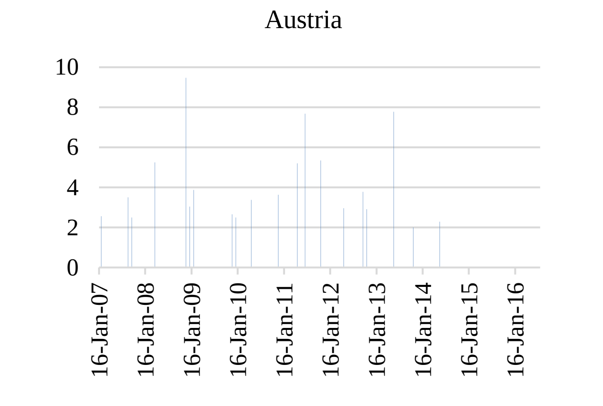
www.ec.europa.eu

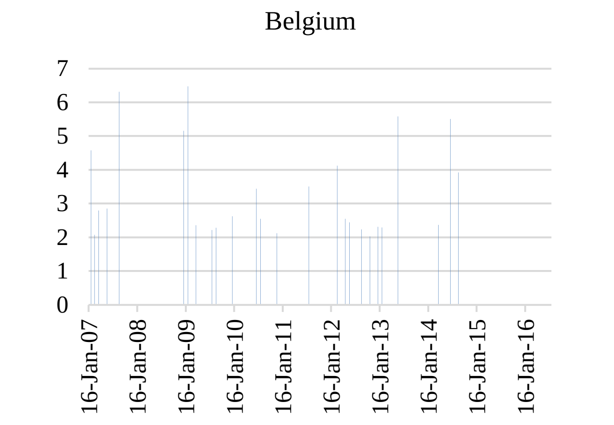
www.markiteconomics.com

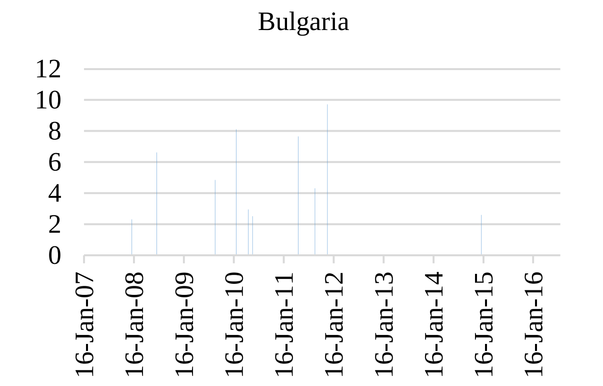
www.bloomberg.com

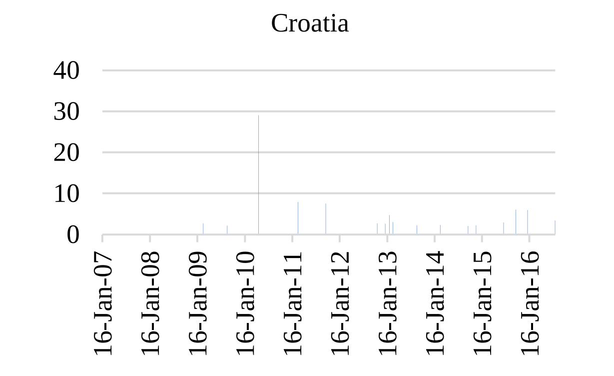
Annex 1

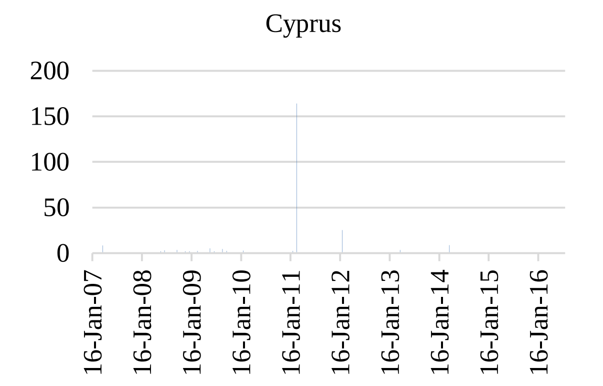
Frequency and Amplitude of Significant Changes in European Stock Markets to Changes in the SentiMent

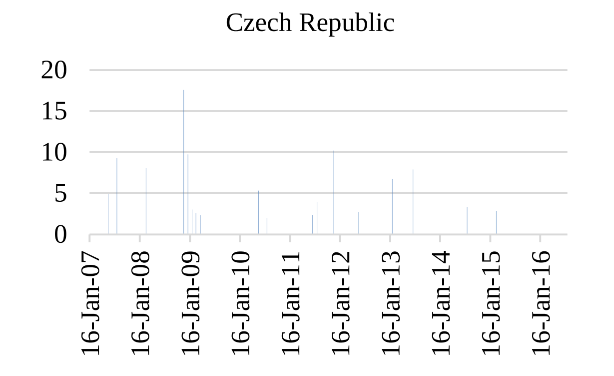


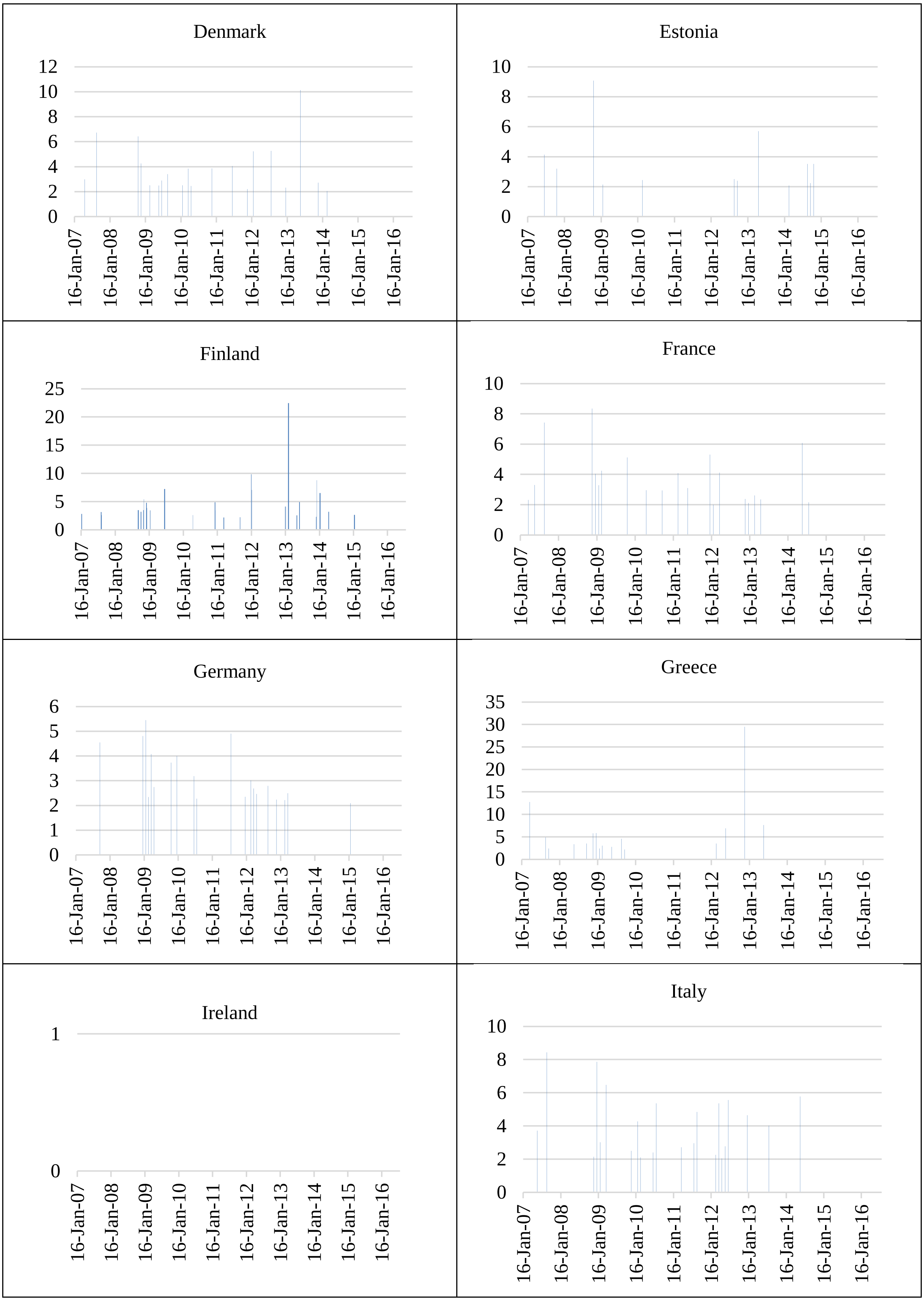








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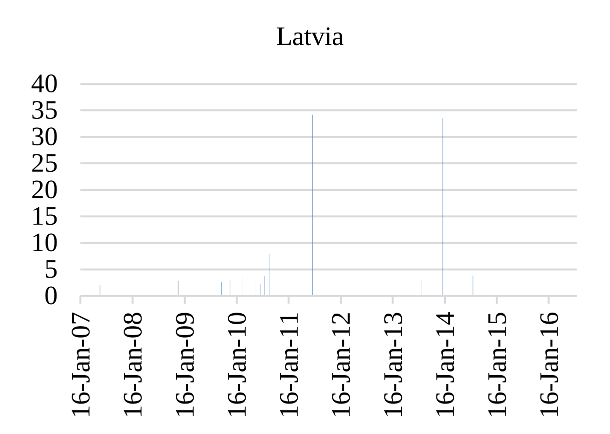
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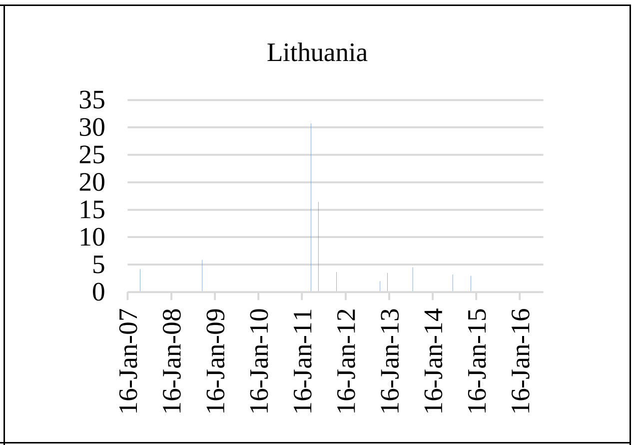
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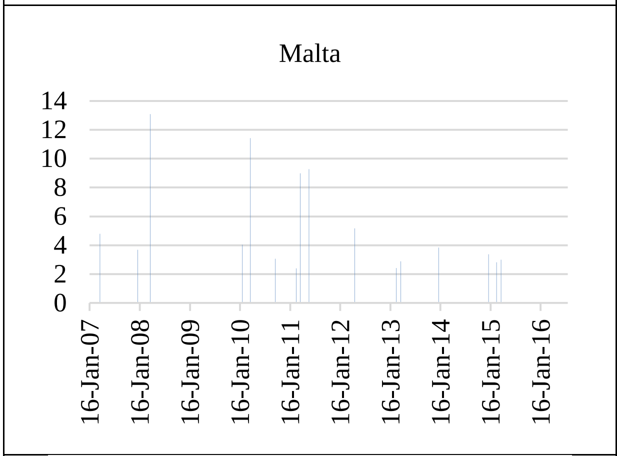
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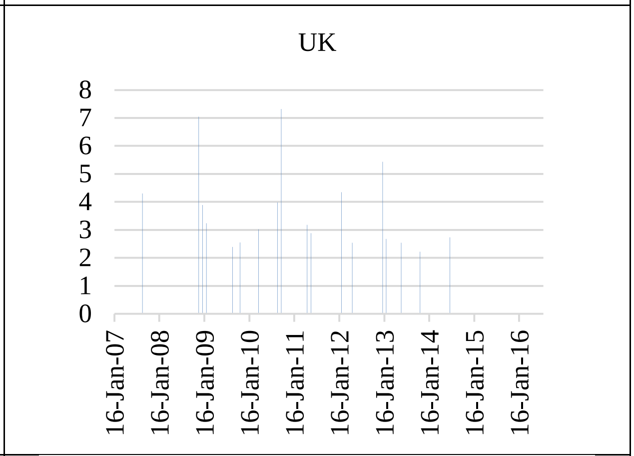
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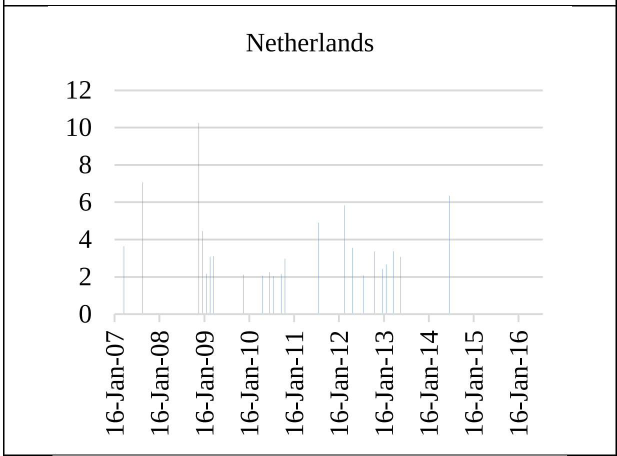
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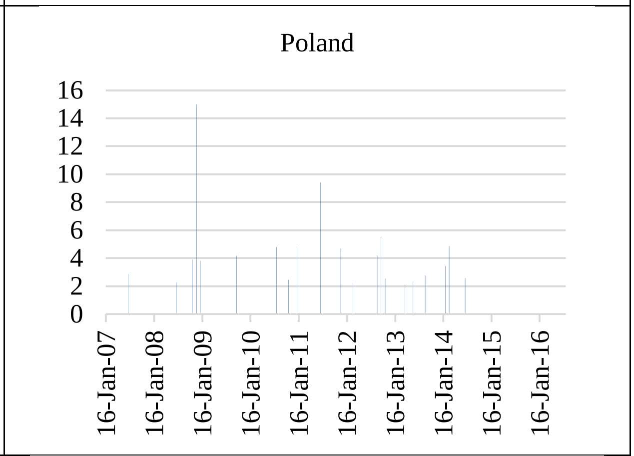
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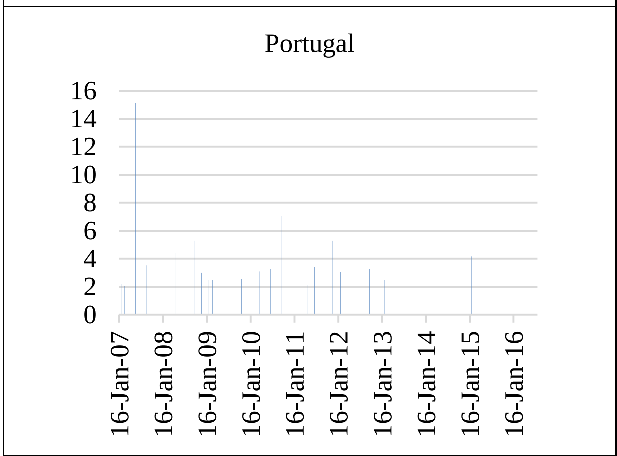
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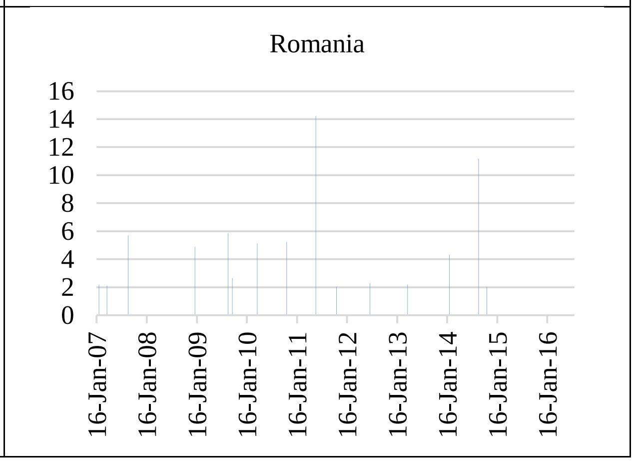
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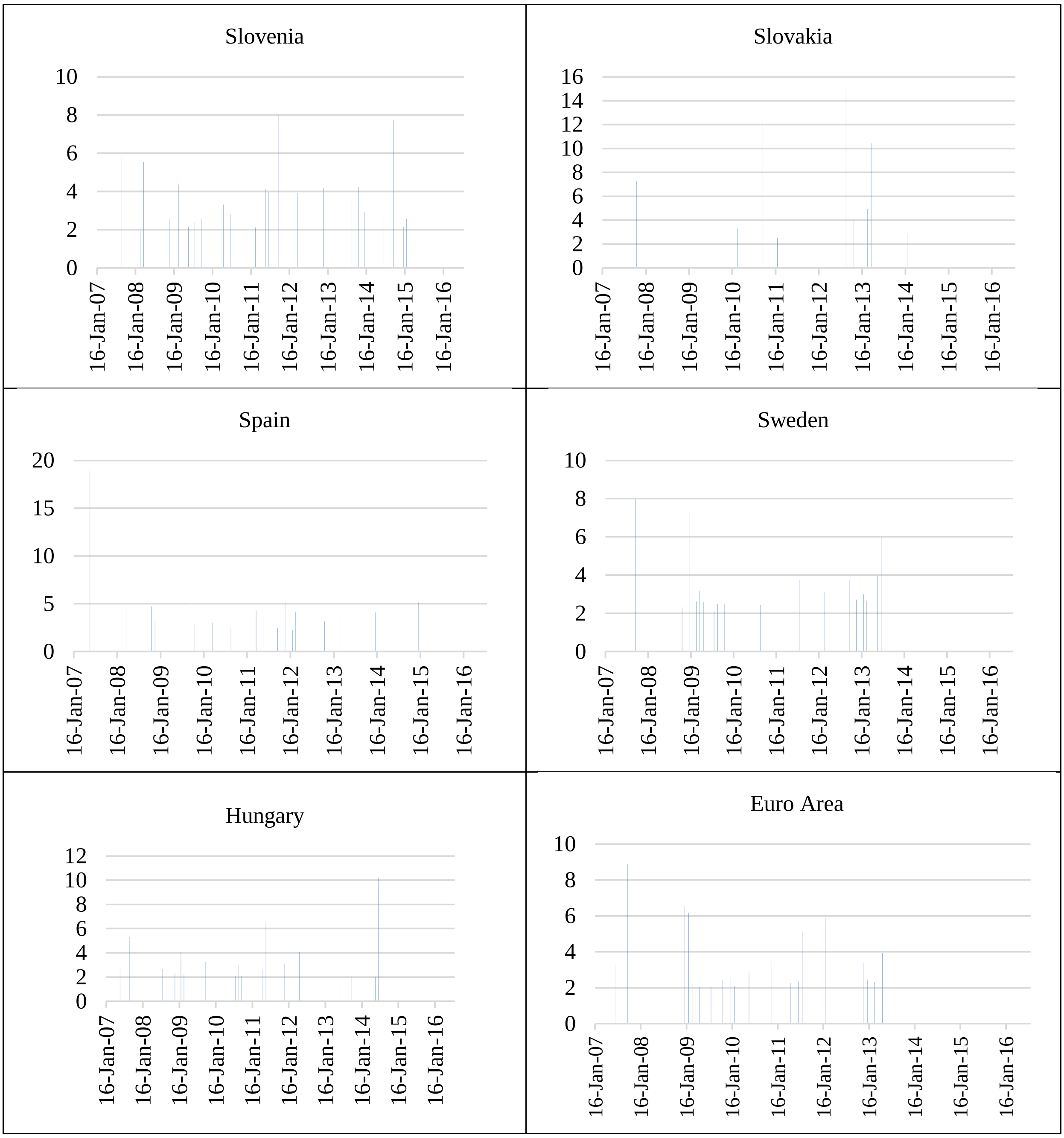
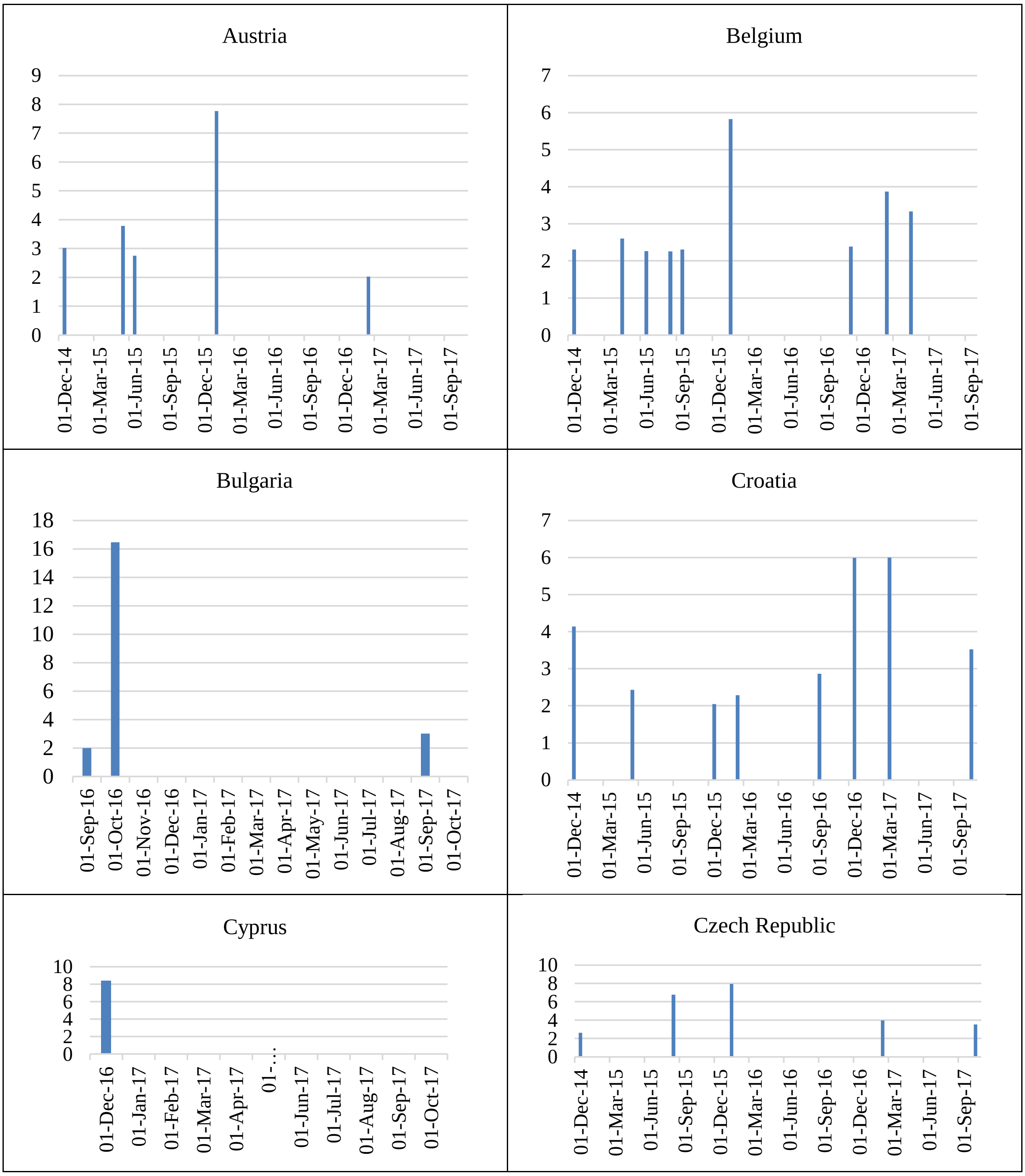
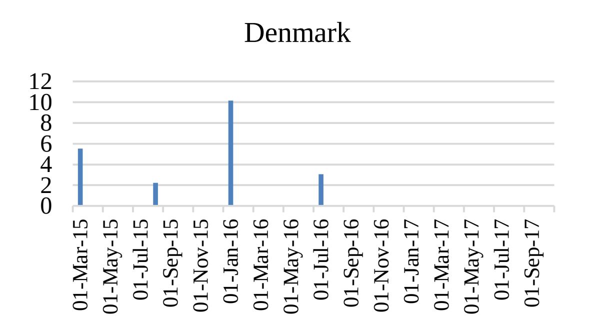


Source: Authors' processing using Bloomberg and European Commission data

Annex 2

Frequency and Amplitude of Significant Changes in European Stock Markets to PMI Changes

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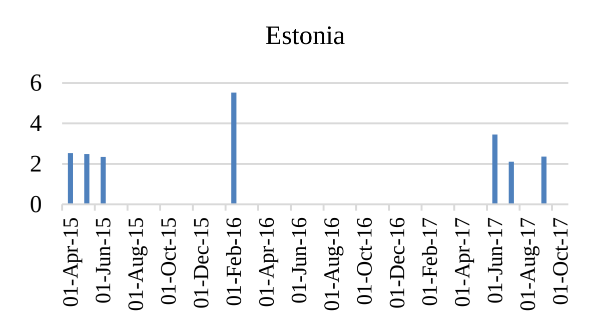
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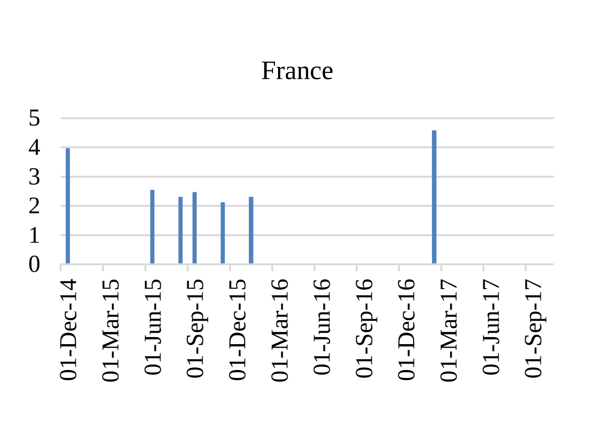
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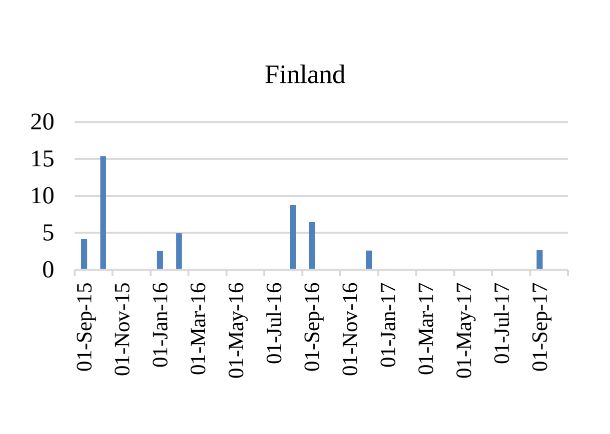
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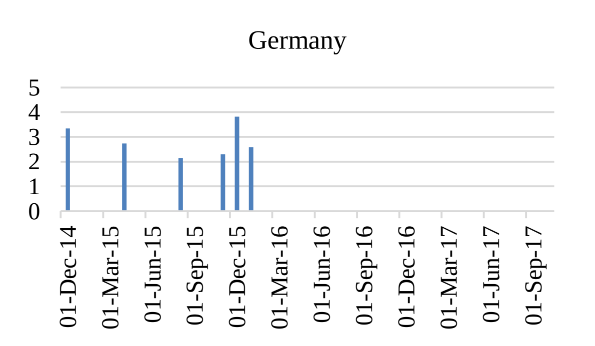
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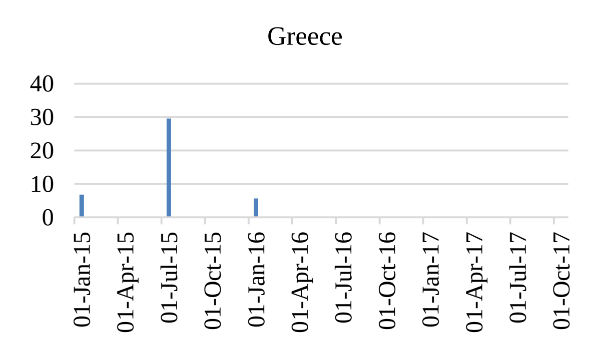
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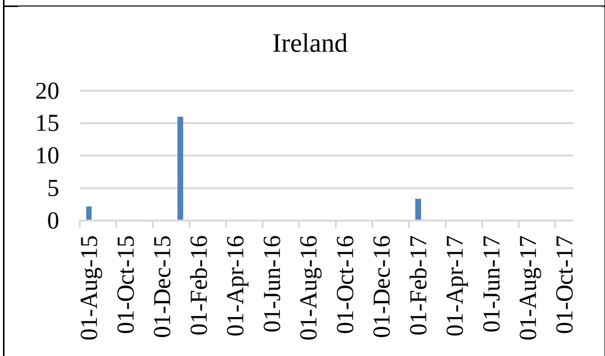
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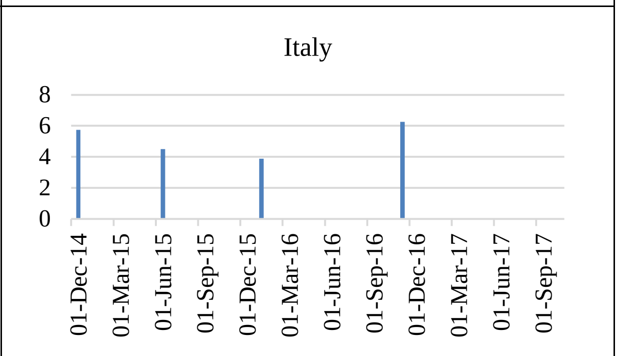
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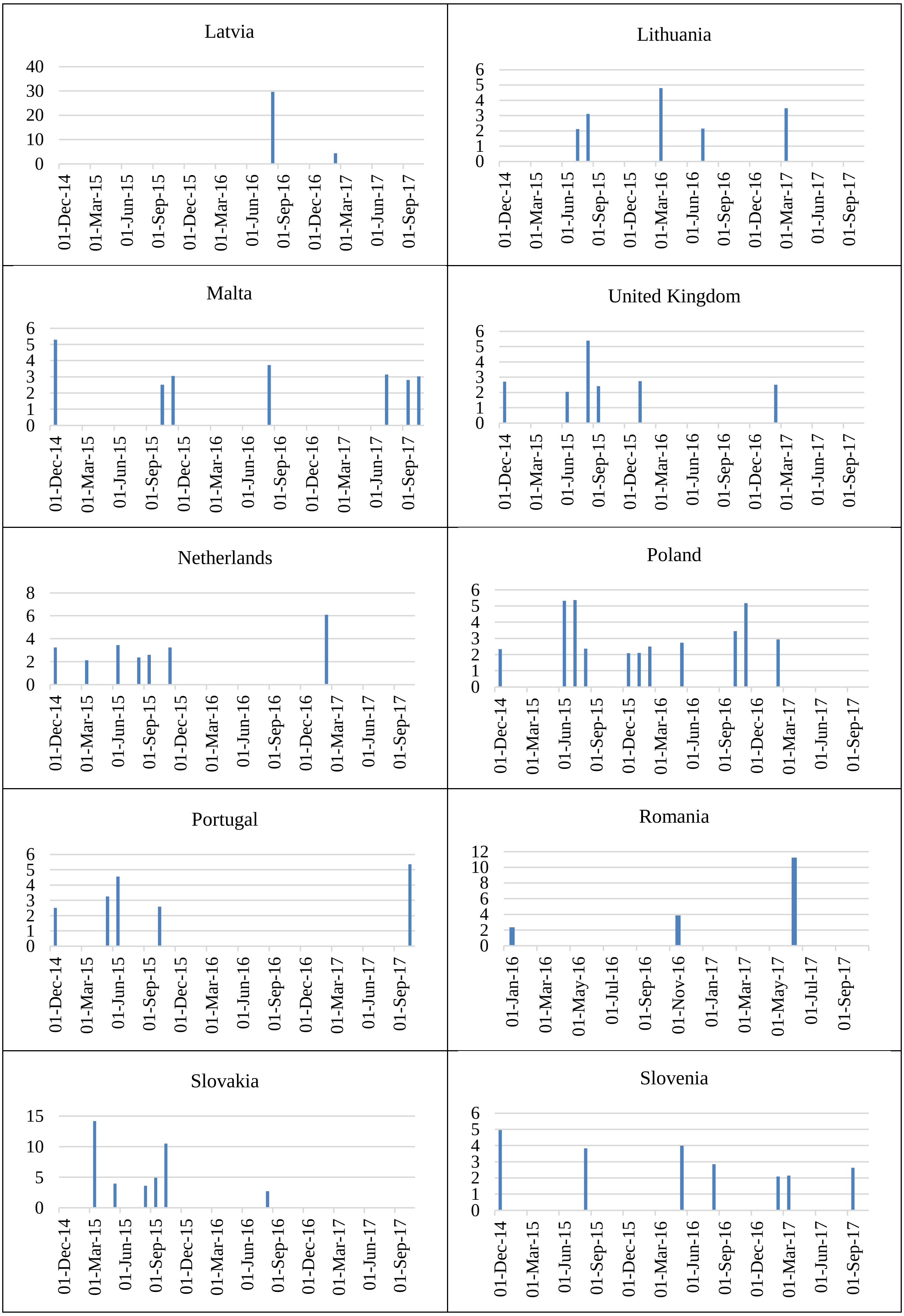
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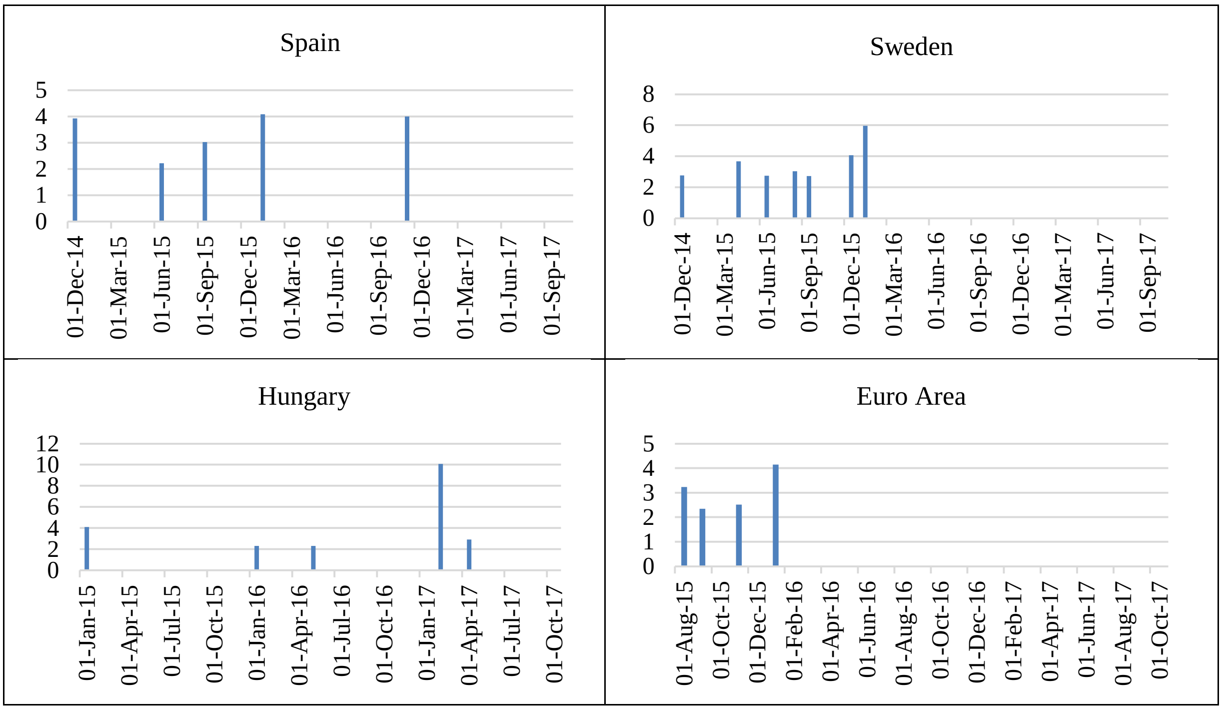

Source: Authors' processing using Bloomberg and Markit Economics data.

INVESTMENT AND THE GOLDEN RULE IN THE EUROPEAN UNION

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Abstract

We will study in this paper the relation between public investment, public debt and fiscal rules in the European Union countries. The strict fiscal rules imposed by EU have negatively affected the investments. The decline in public investment in European Union is related to the fiscal rules (mainly the deficit rule) included in the Stability and Growth Pact (SGP). There have been made several attempts to amend the SGP in such a way to grant a more flexible treatment to capital expenditure when fixing budgetary targets and ceilings. According to the golden rule of budget deficit, investments can be financed through loans, while current expenditure should be financed from taxes. The golden rule promotes thus intergenerational fairness and contributes to economic growth.

Keywords: fiscal rules, public debt, public deficit, Stability and Growth Pact

JEL Classification: E62, H60, E60

1. Introduction

Public investment in the European Union decreased substantially since the beginning of the economic crisis. In most industrialized countries, public investment has been on average below 5 per cent of GDP during the last thirty years, five times lower compared with private investment.

This fall of public investment is a widespread phenomenon, which characterizes not only EU countries, but also many developed economies. Among the factors which explain the decline of investment are structural changes, a general tendency towards a shrinking government sector, and also the need to adjust public

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expenditure in the face of rising public debts (Oxley and Martin, 1991).

The fall in public investment and the current low interest rate environment have made it necessary to stimulate public investment spending with the purpose to increase short-term demand and raise potential output.

The strong fiscal imbalances experienced by most EU countries after the crisis have determined them to adopt new fiscal rules or to implement stricter rules. The Treaty of Maastricht and the European Stability Pact contain clear rules for public debt and for deficits, limiting public debt to 60% of GDP and deficits to 3% of GDP. Public investments can increase only with the condition of satisfying balanced budget rules. As a result of these rules, public investment decreased throughout the European Union.

There have been expressed opinions that the Maastricht convergence process led to a fall in public investment expenditures in EU countries and that the requirements of budgetary discipline within the SGP may limit seriously investment expenditures in EU countries.

Another plan in order to boost public investment was proposed by president Juncker. Nonetheless, the European Fund for Strategic Investments (EFSI), which is fundamental for the new Investment Plan, continues to neglect the negative effects on investments of the strict fiscal rules imposed within EU during recent years.

In national account statistics, investment is defined as expenditures in fixed assets, that is in items that last for more than one year. The most utilised statistical definition of public investment is the gross fixed capital formation of the general government. Fixed assets are not necessarily physical. Intangible assets, like patents or software, enter in the definition of gross fixed capital formation.

2. Literature review

Public investments represent one of the most important instruments for increasing economic growth. Several studies show that public investments have the potential to boost growth not only on short term, but also on long term (Bom and Ligthart,, 2014). Thus, the neglect of public investment will reduce the growth potential of EU economy.

The opinions concerning the relation between public debt - public investments are often divergent. According to some authors (Balassone and Franco, 2000), the obligation to limit the public debt to a certain level has as result a reduction of the public spending for investments. Other studies (Greiner and Fincke, 2009) have shown that a high level of public debt will lead to an increase in demand for public resources necessary for financing the debt service, and this will produce the decrease of the public investments.

Despite the reduction of public investment at the level of European Union countries, existing analyses fail to provide a strong and general indication that public capital is in short supply. Most of the studies analysing the contribution of public capital to production efficiency or growth show that public investment has a positive contribution to countries' productive potential (Easterly, W. and Rebelo, S., 1993).

There are several studies concerning the relationship between public debt, public investment and economic growth.

Peter Diamond (1965) expanded on Samuelson's overlapping generations model to analyse the long term effects of introducing public debt in a neoclassical competitive equilibrium. He did so by introducing production employing a durable capital good into this model. In the model there are used two generations by taking an existing capital stock for granted. Workers work in the first generation and retire in the next generation on capital gains. A constant debt to labour ratio was used in the model because a fixed amount would asymptotically have no effect in a growing economy in the long run. This model was used for showing the possible equilibria and the effects of debt on these equilibria. The Pareto efficient equilibrium was found to be the one in which factors of production, interest on capital and consumption were organized in such a way that interest on capital r is equal to the natural growth rate of labour n.

Elmendorf and Mankiw (1999) discuss what they consider the conventional view of the effects of government debt. According to this view, the issuance of government debt stimulates aggregate demand and economic growth in the short run, because it increases disposable income for households, which has as effect the increase of demand for consumption goods and the increase of aggregate demand for goods and services. National income will go up because of this shift in demand, because the increase in aggregate demand affects the utilization of the factors of production through the

Keynesian concepts of wage rigidity and prices. This positive effect will be even bigger if output is less than capacity and if the central bank will not increase the interest rate as an effect of an expansionary policy.

In the long run the higher budget deficit will have as result a decrease in public savings, which will not be compensated by an increase in private savings. As a consequence total investment will be lower, having a negative impact on GDP due to smaller capital stock, higher interest rate, lower labour productivity and wages.

Delong and Summers (2012) argue that expansionary fiscal policy may be self-financing in the long run in a depressed economy when interest rates are up against the zero lower bound where the central bank is no longer able to perform its stabilizing function because interest rates can't go any lower and there is still a large shortfall in potential output.

Between the papers examining non-linear connections, the paper of Reinhart and Rogoff (2010) is one of the most important. The authors investigated 3,700 annual observations from a database on 20 advanced countries and 24 emerging market economies during 1790–2009. The results of the study are that in the group of advanced economies where the ratio of public debt to GDP was above 90 per cent, median growth (1.9 %) is 0.9–2.0 % points lower over the whole period than in the group of countries with a lower debt burden (with a debt ratio of 0–30, 30–60, and 60–90%). They also found that average growth in economies with higher debt levels is 1.3–2.0 percentage point lower (1.7%). The gap was even wider in the group of emerging economies. For the period 1900–2009, median and average growth (2.9 and 1.0%) was 1.5–1.6 percentage points, and 3.1– 3.3 percentage points lower in countries with a debt/GDP ratio above 90% than in economies with public debt of 0–30, 30–60, and 60–90%. A common feature of the findings across both advanced and emerging economies was that there was a sharp fracture at the 90 per cent threshold and the results suggested a general correlation between growth dynamics and public debt.

Afonso and Gonzales (2011) analysed the influence of the budget components - the categories of expenditure and income on the economic growth in EU 15, during 1971-2006. The study reaches the conclusion that public investments have a positive impact on economic growth.

Checherita and Rother (2010) studied the relationship between public debt and economic growth in the Euro Zone and demonstrated the existence of a non-linear, concave relationship between these two variables, which has the turning point of 90% - 100% of the GDP. The study shows that high levels (over 90% of GDP) and increasing public debt influence economic growth due to the increase of the long term interest rate that has a negative impact on private investments.

3. Fiscal rules in the European Union

Some authors (Blanchard and Giavazzi, 2004) propose to modify the Stability and Growth Pact so as to exclude public investment spending completely from the measure of fiscal deficit that is subject to the rule. These types of arguments start from the idea that the Stability and Growth Pact (or any other similar deficit rule) is intrinsically discriminating against public investment and the only solution would be their exclusion from the fiscal deficit rule in order for public investment to regain their optimal level.

The fiscal rules included in the Stability and Growth Pact have as purpose to ensure an efficient coordination of budgetary policies of different Euro zone countries.

These rules are centred around an objective of structural budgetary balance - MTO (Medium Term Objective) which must be reached and maintained on medium term. This medium term objective must let automatic stabilisers act within the cycle: the real budgetary balance fluctuates depending on the cycle around its fundamental tendency centred on MTO.

The modality of calculus of actual MTO is based on the criterion of public debt sustainability according to which the actualised sum of primary surpluses is superior or equal to the public debt.

Governments make debt for financing public investments projects in addition to private investment, all of which have as result a bigger economic growth. An appropriate deficit and debt levels are also necessary conditions for growth. The following criteria define healthy public finances:

1) Comparison between revenue and public expenditure by means of a definition of public deficit, which tends to zero at the optimal level

$$D_t^{PN} = (G_t - T_t) + I_t, I_t = i_t B_{0,t-1} \tag{1}$$

where D^{PN} is the nominal budget deficit, G_t is public spending, T_t is the public income, I_t is the t volume of interest paid, i_t is the nominal effective rate of interest, and $B_{0,t-1}$ is the total value of domestic public debt from the period 0 to period t-1.

2) Compliance with the inter-temporal budget constraint

$$T(t) + \frac{dB(t)}{dt} = G(t) + r(t)B(t) \tag{2}$$

$$\frac{dB(t)}{dt} = r(t)B(t) + G(t) - T(t) = r(t)B(t) - S(t) \tag{3}$$

where t represents time, r is the interest rate and S describes the primary surplus calculated as difference between primary income and expenses for goods and services without taking into account the payment for interest. The first of the above equations shows that income from taxes and new issued debt instruments must be equal with governmental expenses. The reorganisation of the first relation generates the second which shows that the change in debt is equal with the sum between the payments of interest on existent debt instruments and primary deficit.

3) Following Blanchard (Blanchard et al., 1990), a comparison between the rate of economic growth and the interest rate that is paid for the debt should be considered:

$$db/ds = g + h - t + (r - \Theta)b = d + (r - \Theta)b \tag{4}$$

where b is the ratio of real debt on GDP while s refers to time, g represents government spending on goods and services, h refers to transfers, t is for taxes, r is the real interest rate and θ is the rate of economic growth. Blanchard starts from the supposition that the real interest rate exceeds the growth rate, that is $r - \Theta$ is positive.

Thus, fiscal policy is sustainable if the real debt does not grow faster than the interest rate (or if the ratio of real debt to GDP does not grow faster than the excess of the interest rate over the growth rate).

4. The golden rule

In order to support public investment a different fiscal policy would be necessary at the level of the European Union. In this direction, one proposal was the implementation of the golden rule of public investment, as developed by the economist Richard A. Musgrave. This rule states that net public investment (gross public investment minus depreciation), that is increases of the public and/or social capital stock providing future benefits should be financed by debt and consequently excluded from balanced-budget rules.

The golden rule of public sector borrowing states that government borrowing should not exceed public capital formation over the cycle. This rule has been proposed as a way of modifying and loosening the EMU fiscal rules. There have been expressed opinions that the Stability and Growth Pact in its initial version may reduce the public sector's contribution to capital accumulation, while implementation of the golden rule may prevent an investment slowdown in the public sector of EMU member countries. After the change of the Stability and Growth Pact, only public investment can justify the exceeding of the maximum value of annual government budget deficit of 3% of GDP.

According to this rule, net public investment could be financed by government deficits, which promotes intergenerational fairness and economic growth. The investments are financed by future generations through the debt service. If future generations do not contribute to financing investments, this will lead to a disproportionate burden for the present generation, through higher taxes or lower spending, creating incentives for the under-provision of public investment to the detriment of future generations. There is evidence that this under-provision has indeed been characteristic of periods of fiscal contraction – not only during the current crisis, but also in relation to the decline in public investment observed during previous crises (Turrini, 2004).

Usually decisions concerning government investment expenditures are made by trading-off efficiency objectives (how much investment is needed to adapt the supply of infrastructures and other public-purpose capital assets to the needs of the economy) and budgetary objectives (which is the amount of investment expenditure consistent with the target budget balance).

Fiscal rule are used for modelling budgetary objectives, and the desired budget balances are represented as a function of output gaps, debt levels and past budget balances. In such a framework, the presence of the EU fiscal framework is assumed to potentially modify the parameters of the fiscal rules, the reaction of fiscal authorities to output gaps, debt levels and past budgets (Gali and Perotti, 2003).

According to the golden rule, fiscal policy should have as purpose a stable allocation of public sector resources during a business cycle. The increase of government borrowing has as consequence the increase of the real interest rate which results in crowding out investment. Therefore, capital accumulation fails, and this has a negative impact upon economic growth.

The golden rule states that over the economic cycle, the government will borrow only to invest and not to fund current spending. Therefore, over the cycle the current budget must balance or be brought into surplus.

The golden rule allows net borrowing by the government to finance public investment, and current spending to be financed out of current revenues. Temporary net borrowing for cyclical stabilisation purposes could also be allowed, as long as such cyclical fiscal deficits are matched by surpluses in cyclical upturns so that net borrowing for stabilisation purposes averages zero over the entire business cycle.

A possible objection to the adoption of a Golden Rule is that it can undermine debt sustainability. At the moment, the strictest fiscal rule at EU level in normal times is the medium- term objective, i.e. a structural deficit of 0.5 % of GDP or less.

The implementation of the Golden Rule for Public Investment could be realised provided the European Commission and the European Council could use the actual interpretational leeway to change the rules regarding the SGP. There are some elements in EU legislation which can justify the Golden Rule. The Article 126 TFEU indicates the European Commission to ‘take into account whether the government deficit exceeds government investment expenditure’ within the report on the existence of an excessive deficit. The investment clause in the Stability and Growth Pact introduced in 2013 also permits temporary deviations from structural objectives, complying with some very restrictive conditions. There are also several commonly agreed exceptions (especially in the case of the new debt rule) and unclear specifications (the method to be used for estimating the structural deficit).

Another possibility would be to use the provision concerning a severe downturn in EU in order to allow a deviation from the consolidation mechanisms. Thus an European Investment Programme should be implemented. The Commission has explicitly made a comparison with the 2008 European Economic Recovery Plan to give an example of the potential use of this provision (European Commission, 2015: 17). The utilisation of this provision ‘should remain limited to exceptional, carefully circumscribed situations to minimise the risk of moral hazard’ (European Commission, 2015:17). It may be sustained that the Euro area is currently in precisely such an exceptional situation after several years of recession.

5. Conclusions

The Golden Rule supports public investment as an essential element of public spending. Unlike the Juncker Plan, it provides a direct boost to public investment on the national level.

The Golden Rule is a fiscal policy tool having as purpose to protect public investment in the medium term and cannot contribute to the economic recovery in EU very quickly. Therefore, besides the application of the rule, it should be necessary a short-term European Investment Programme similar to the European Economic Recovery Plan adopted during the financial crisis.

Such a program could help to increase public investment up to the proposed level with the implementation of the Golden Rule. This program could also contribute to a broader definition of public investment, beyond the mere definition from the national accounts. New investments could include education, but also spending in order to realize some goals from the strategy Europe 2020, like social inclusion and other fields which were affected by the austerity policies. This program and the application of the golden rule could contribute to re-launching the European economy.

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USING THE SYMMETRIC MODELS GARCH (1.1) AND GARCH-M (1.1) TO INVESTIGATE VOLATILITY AND PERSISTENCE FOR THE EUROPEAN AND US FINANCIAL MARKETS

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Abstract

In this paper, we used the GARCH (1,1) and GARCH-M (1,1) models to investigate volatility and persistence at daily frequency for European and US financial markets. In the study we included fourteen stock indices (twelve Europeans and two Americans), during March 2013 - January 2017. The results of the GARCH (1.1) show that the models are correctly specified for most of the analysed series (except for the WIG30 index). The study found that the BET-BK index recorded the lower persistence of volatility, meaning that the conditional volatility tends to revert faster to the long-term mean than the other stock indices analysed. In the case of the GARCH-M (1.1) model, the variance coefficient in the mean equation was statistically significant and positive (thus confirming the hypothesis that an increase in volatility leads an increase in future returns), only for six of the analysed series. The strongest relationship was recorded for the US index, S&P500. It is also recorded for the Romanian stock indices: BET and BET-BK. For the BET index, the conclusions are in line with the results of previous studies.

Keywords: stock market, volatility clustering, volatility persistence

JEL Classification: C22, C32, C51, G11, G17

1. Introduction

The global financial crisis has made financial markets characterized by a high degree of uncertainty and high volatility in the prices of financial assets, irrespective of their type: stocks, bonds, commodities, derivatives, etc. Volatility makes difficult the anticipation

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of the future evolution of earnings from financial placements and requires increased attention from investors, speculators, fund managers and, last but not least, financial market regulators.

If the return, the risk, the time horizon, and the liquidity of financial placements are notions with which stock market investors are largely familiar, volatility is a more difficult variable to quantify, as it cannot be directly observed. Financial market participants perceive volatility differently, depending on the daily variation in trading prices (decrease or increase). Volatility has been shown to increase as the market recorded significant declines in financial asset prices and is lower when the market is on the rise. As a result of this, the volatility is usually associated by investors with the loss rather than profit, and in this case, they approach with caution the periods of increased volatility. Instead, speculators step up their trading activity during these times, attracted by increased profit opportunities. The volatility behaviour can be analysed through the variation in the return on financial assets. Studies on financial time series have highlighted some of their features such as leptokurtotic distribution, leverage effect, heteroscedasticity, fat tails, volatility clustering, autocorrelation or serial correlation in residuals, etc. The phenomenon of "volatility clustering" visible effect of heteroskedasticity was first observed by Mandelbrot (1963). He concluded that high return variations are followed by major future changes, while low return variations are most likely followed by small fluctuations. Volatility clusters can be observed by analysing the volatility chart of stock indices included in this study. We test the GARCH (1.1) and GARCH-M (1.1) models and analysed fourteen stock indices: twelve Europeans indices (less founded in the specialty studies, including BET and BET-BK) and two Americans indices.

The study period is a more recent one, March 2013-January 2018, and it should be characterized by a lower volatility than the one recorded during the financial crisis.

The selection of the two GARCH (1.1) and GARCH-M (1.1) models is motivated by the conclusions of previous studies on this theme. Hansen and Lunde (2005) showed that a GARCH (1.1) model using only three parameters in the conditional variance equation is sufficient to model the financial series. The study of the applicability of the GARCH-M (1.1) model on this set of stock indices was based than on observation of a positive relation between the assumed risk and the obtained return, relation which can be surprised by a variable introduced in the mean equation of the model. This variable must be positive and statistically significant. A previous study, conducted over the period 1997-2012, for the BET index (which is found in this study) reported the absence of this relationship regardless of the frequency of the data analysed. In addition, the model failed to remove the ARCH effects left of the daily residuals series.

The originality of the study is given both by the analysed period of time (March 2013 - January 2018) and the stock market indices studied. Most of the indices (with the exception of S&P 500, Dow Jones Industrial 30 and DAX30) are from European Union countries (except Switzerland). Some are neighbouring countries (investor behaviour should be similar) but the common feature is that they have not yet adopted the euro (transactions in the national currency, foreign investors thus assuming, besides market risk and foreign exchange risk in the moment of making investments on the capital markets of these countries).

Another element of originality is that the stock indexes analysed are less well-researched in the previous studies, the reason for the exclusion being that some of the capital markets are small size and thus the interest of the foreign investors is lower.

2. Literature review

Forecasting volatility (volatility perceived as a source of risk by investors) has constituted a subject of study for the international scientific community. In time, a lot of models of volatility study have been proposed and tested for various time series and different frequencies.

The first model of volatility estimation was Black and Scholes (1975, pp. 307-324) for implicit volatility in options, followed by the ARMA model proposed by Box and Jenkins (1976) used to study the volatility of financial assets. These models were based on the assumption that the price series of the financial assets have a constant variance, the hypothesis that proved to be erroneous. Previous models cannot capture the stylized facts (Cont, 2001) of the financial returns such as: volatility clustering, leptokurtosis, leverage effect, fat tail, etc.

The ARCH (Autoregressive Conditional Heteroscedasticity) is a model proposed by Engle (1982), in which the variation depends on the previous patch errors, seemed to solve the above problems. The basis of the model was the empirical observations of the change in time of volatility and the fact that it depends on its previous values. But there was another problem, that the coefficients of the ARCH model are hard to estimate. Four years later, Bollerslev (1986), proposed an improved form of ARCH, namely GARCH (Generalized Autoregressive Conditional Heteroscedasticity). Empirical observations have shown that financial time series do not usually have a normal distribution (assuming skewness 0 and kurtosis 3) and rather a leptocurtotic one.

These observations underlie the leverage effect (the effect that news has on volatility) first presented by Black (1976). It has been noticed that negative news has a stronger impact on volatility than positive ones. The GARCH (1.1) fails to capture the leverage effect and so it was necessary to develop extensions of this model such as EGARCH, TGARCH, GARCH-M, etc. In the case of financial investments, assuming an increased risk is associated with a high expected return.

To capture the relationship between the expected return and the associated risk of a financial asset, Engle, Lilien and Robins (1987) expanded the GARCH model, introducing a new term, the conditioned volatility, in the mean equation of the classical model. All of these models have been tested on different markets and financial assets over different periods of time and on different frequencies (daily, weekly, monthly) and their conclusions varying. Akigray (1989, pp. 55-80) tested ARCH (2), EMWA and GARCH (1.1) to identify the time series properties of US expected stock return. The conclusion of the study was that GARCH (1,1) is the most appropriate model. Pagan and Schwert (1990, pp. 267-290) concluded that the EGARCH model is more performing than nonparametric models.

Cao and Tsay (1992, pp 165-185) supported the EGARCH model providing the best predictions for low capitalization shares. The study by Sill (1993, pp. 15-27) concluded that the volatility of the S&P500 index is higher in times of recession than in economic expansion and that spreads between corporate bond rates and government bonds predict future stock market volatility. Donaldson and Kamastra (1997, pp. 17-46) found that the persistence of volatility effects in European and North American markets is lower relative to Japanese market. Franses and Djik (1998, pp. 229-235) compared volatility predictions of QGARCH (1.1), GJR-GARCH (1,1), GARCH

(1,1) and Random Walk for stock indices in Spain, Germany, Italy, Netherlands and Sweden.

Nam, Pyun and Aruza (2002, pp. 563-588) applied the GARCH-M model for US stock indices during the period 1926-1997. They concluded that negative returns on average reverted more rapidly to long-term mean than positive returns.

Harq et al. (2004, pp 19-42) tested Random Walk, ARMA and GARCH-M for ten African and Middle East markets.

Selcuk (2004, pp. 867-874) surprised the persistence of volatility effect in emerging markets. Caiado (2004, pp. 3-21) investigated mean reversion behaviour for the Portuguese market (using the PSI20 index) and found that the mean reversion is recorded for low frequency and not for high frequency data.

Lupu (2005) demonstrated that GARCH model captures the characteristics of the Romanian capital market. Two years later, Lupu and Lupu (2007) used the EGARCH model to investigate the same capital market.

Rizwan and Khan (2007, pp. 362-375) have surprised the phenomenon of volatility clustering on the Pakistan market. Magnus and Fosu (2006, pp. 2042-2048) found a high level of persistence for the Ghana capital market.

Tudor (2008, pp.183-2008) tested the GARCH and GARCH models for the main indices of the American and Romanian financial markets. The GARCM-M model performed better and revealed the correlation between volatility and expected returns on both markets.

Panait and Slăvescu (2012) investigated the applicability of GARCH-M (1,1) on the Romanian capital market (1997-2012) for low and high-frequency data. The results of the study were in line with those of Caido (2004). In the Panait and Slăvescu' study, the mean reverting was ascertained for low frequency data (weekly and monthly) and less for high frequency data (daily). But, GARCH-M (1.1) "failed to confirm (...) the theoretical hypothesis that an increase in volatility leads to a rise in future returns, mainly because the variance" (Panait and Slăvescu, 2012, pp. 55).

The study of the Romanian capital market was continued by Miron and Tudor (2010). The paper focused on asymmetric GARCH, EGARCH, PGARCH, TGARCH) with a daily data frequency. For the model errors were used different distributions (normal distribution t, GED distribution and t student). The conclusion was that the

EGARCH (with Student and GED errors distributions) best surprised the characteristics of returns for Romanian capital market.

EGARCH model was best evaluated in the estimation of exchange rate volatility and stock indices and by other authors such as Lee (1991), Heyen and Kat (1994).

We will continue to investigate the applicability of the GARCH and GARCH-M models for a more recent period of time, March 2013 - January 2018, and for a number of stock indices little found in previous studies.

The main objective is to discover the current characteristics of capital markets, which would be a useful tool for all investors to substantiate the investment strategy.

3. Data and research methodology

As mentioned above, in our research we included fourteen stock indices: twelve in Europe and two of the main US stock indices (Dow Jones Industrial Average and S&P 500).

The indices and the number of daily observations for each index can be found below (Table 1).

Table 1

The stock indices included in the study

Symbol	Index name	Country	Nr obs.
DAX	Deutscher Aktien IndeX 30	Germany	1248
GSPC	The Standard & Poor's 500	US	1240
DJI	The Dow Jones Industrial Average	US	1240
BET	Bucharest Exchange Trading	Romania	1233
BETBK	Bucharest Exchange Trading Benchmark Iindex	Romania	1233
BGTR30	*BG TR30 Index*	Bulgaria	1215
BUX	Budapest Stock Exchange Index	Hungary	1226
CROBEX	The Croatia Stock Market	Croatia	1226
FTSE	The Financial Times Stock Exchange 100 Index	England	1244
KAX	KAX All-Share Index	Denmark	1228
OMX30	The OMX Stockholm 30	Sweden	1236
PX	Prague Stock Exchange *Index*	Czech Rep.	1229
WIG30	Warsaw Stock Exchange *Index*	Poland	1227
SMI	The Swiss Market Index	Switzerland	1235

Source: Yahoo Finance and Investing.com. Calculations by the authors

The time series of the fourteen stock indices are adjusted to corporate events (dividends, capital increases, consolidations, etc.) according to their calculation methodology.

Considering that the purpose of our analysis was not to study the correlation between these stock indices, there was no need for the perfect chronological synchronization of the data series analysed.

The study period is March 2013-January 2018, the frequency of the data is daily.

Price ranges were obtained from https://finance.yahoo.com for US stock indices and www.investing.com for European stock indices.

With the exception of DAX30 (which is expressed in EUR), all other stock indices are expressed in the national currency of those countries. In Figure 1 of the annexes we have graphical representations of the initial time series included in the study.

The price series were subsequently transformed into series of logarithmic returns, resulting in a database of fourteen logarithmic returns series.

Table 2

Descriptive statistics for the returns series

	Mean	Maxim	Minim	Std. Dev.	Skew	Kurtosis	Jarque-Bera	P-val
DAX	**4.4814**	479.69	-699.8	**115.15**	-0.344	5.426	320.67	0
SPX	**1.0635**	70.02	-72.36	**13.79**	-0.460	5.842	460.81	0
DJI	**8.7314**	619.07	-610.32	**125.63**	-0.331	5.204	267.05	0
BET	**1.8882**	213.73	-461.58	**53.19**	-0.887	10.907	3309.3	0
BET-BK	**0.4559**	43.47	-81.81	**9.0251**	-1.122	13.522	5836.2	0
BGTR30	0.2377	20.62	-19.79	2.7214	-0.186	11.574	3713.8	0
BUX	**16.279**	1281.72	-1341.9	**240.84**	-0.219	5.4846	321.01	0
CROBEX	-0.084	47.68	-62	10.32	-0.544	8.2089	1427.8	0
FTSE	0.9642	219.67	-288.78	55.82	-0.170	4.9700	201.55	0
KAX	0.5853	52.07	-55.05	10.50	-0.413	6.2901	580.17	0
OMX30	0.3138	55.270	-114.63	14.58	-0.492	7.3030	982.51	0
PX	0.0797	43.28	-45.68	8.39	-0.398	5.4003	322.56	0
WIG30	0.3536	79.25	-141.65	24.83	-0.312	5.3872	307.07	0
SMI	**1.6085**	289.9	-797.59	**79.23**	-1.178	13.638	5985.9	0

Source: Yahoo Finance and Investing.com, calculations by the authors

From Table 2 we draw the next conclusions:

All indices had an upward trend, except for the Croatian stock exchange index. For all the time series the value of standard deviation is larger than the mean values.

All data series present negative asymmetry, excess kurtosis and fat tail, indicating leptocurtotic distributions. The deviation from normality is more pronounced in the case of the SMI index (in Switzerland), with a skewness (-1.178) and a kurtosis (13.6385), values being far from those of the gaussian distribution (skewness 0 and kurtosis 3). The same characteristic can be observed for the Romanian indices: BET-BK and BET.

None of the time series are normally distributed, as proven by values for the Jarque-Bera tests (Table no. 2).

We continued to perform tests to determine heteroscedasticity and volatility clustering. The analysed series of returns show the phenomenon of volatility clustering, a phenomenon considered to be a consequence of the leptokurtotic distribution. This is the tendency of very high or very low volatile volatility periods to group together. The explanation for this phenomenon is that abnormally large shocks occurring during the current period will cause an immediate increase in volatility, and this will also rise in the next period, depending on investors' perception of the intensity of these shocks.

For the investigation of heteroscedasticity, we calculated the autocorrelation (AC), the partial autocorrelation (PAC) and applied the Q test (Ljung-Box statistic), the results being centralized below (Table 3). In our calculations, we used a 20 period lags. We can see that most data series present serial correlation till the 20-th lag (the Q test being significant at 10%), thus confirming the presence of heteroscedasticity. We also have three exceptions: DJI, BET and BET-BK for which the probability associated with the Q test does not allow us to reject the null hypothesis, the lack of the serial correlation till the 20-th lag.

Table 3
Estimation of autocorrelation, partial autocorrelation and Q-test with 20 lag

	lag	AC	PAC	Q-Stat	P-val.
DAX	20	0.046	0.053	35.628	0.017
SPX	20	0.058	0.057	31.148	0.053
DJI	20	0.060	0.053	21.104	0.391
BET	20	-0.038	-0.041	17.697	0.607
BETBK	20	-0.023	-0.027	18.318	0.566
BGTR30	20	0.031	0.028	36.911	0.012
BUX	20	-0.022	-0.024	33.036	0.033
CROBEX	20	0.028	-0.002	126.94	0.000
FTSE	20	0.036	0.040	34.994	0.020
KAX	20	-0.021	-0.030	29.279	0.082
OMX30	20	0.055	0.049	53.515	0.000
PX	20	-0.055	-0.053	32.887	0.035
WIG30	20	0.005	0.015	31.058	0.054
SMI	20	-0.009	-0.013	37.493	0.010

Source : Yahoo Finance and Investing.com, calculations by the authors

In conclusion, we found heteroscedasticity in returns for only eleven of the fourteen series studied. As heteroscedasticity is a precondition for applying GARCH models, it is possible we cannot calibrate these models for the three series of returns (DJI, BET and BET-BK). After we discovered the presence of the phenomenon of volatility clustering and heteroscedasticity, we passed to the estimation of the parameters of GARCH (1.1) and GARCH-M (1.1) for all fourteen datasets.

As previously mentioned, the GARCH model (1.1) was proposed by Bollerslev (1986) and has two equations, one for the mean and one for the variance of the time series presented below.

The mean equation: $Ri=\mu+\varepsilon_i$ (1)

The variance equation: $\sigma i^2=\omega+\alpha\varepsilon^2{}_{i-1}+\beta\sigma^2{}_{i-1}$ (2)

Where: ω is the mean, $\varepsilon^2{}_{i\_1}$ is the term ARCH (the last volatility information measured as lag of the squared residuals of the mean equation), $\sigma^2{}_{i\_1}$ is the term GARCH (the forecast variance of the previous period). We observe that the conditional variance (the variance of the next period calculated on the basis of the previous values) is a function of three variables: the mean (ω), the term ARCH and the term GARCH. The persistence of conditional volatility is given by the sum of ARCH and GARCH coefficients and it must be subunit ($\alpha+\beta<1$). This is an essential condition for a mean reverting process.

The GARCH-M (1.1) proposed by Engle, Lilien and Robins (1987) is an extension of the GARCH (1.1) model and has the following equations:

The mean equation: $Ri=\mu+\beta i\sigma i^2+\varepsilon_i$ (3)

The variance equation: $\sigma i^2=\omega+\alpha\varepsilon^2_{i-1}+\beta\sigma^2_{i-1}$ (4)

Unlike the initial model, GARCH (1.1) GARCH-M (1.1) has the term β_i representing the volatility of the analysed assets in the mean equation. It should capture the positive relationship between the assumed risk and the expected future return of this placement. One of our study objectives was to investigate the existence of this relationship for our set of stock indices between March 2013 and January 2018.

4. Results and interpretations

In table 4 (found in Annexes) are presented the values for the coefficients: ω, α and β of the GARCH (1.1) model. In estimating model for each of the fourteen series we started from the assumption that the errors are normally distributed. Analysing the data presented in the table we can conclude that all coefficients of the variance equation (ω, α and β) are statistically significant for all data series at a high confidence level, 99%. It had high values for z-statistical and low p-value. The estimated coefficients of the model fulfil the condition that $\alpha + \beta<1$, a condition necessary for the process to be mean reverting. If $\alpha + \beta> 1$, the process would be an explosive one, and the modelling of the data series would have to be done with another GARCH model (the IGARCH model).

We can conclude that the most time series (except WIG30) are mean reverting. To investigate the return to average volatility behaviour (more precisely the persistence of volatility) we calculated the sum of the coefficients α and β (Table 5).

Table 5

The persistence value in the GARCH (1,1)

Indices	Persistence
DAX	0.986006
SPX	0.915459
DJI	0.878765
BET	0.906898
BET-BK	0.805244
BGTR30	0.950867
BUX	0.938798
CROBEX	0.932255
FTSE	0.931947
KAX	0.983791
OMX30	0.975901
PX	0.953836
WIG30	-0.18871
SMI	0.973519

Source: calculations by the authors

It can be noticed that the conditioned volatility of returns for BET-BK tend to revert fastest to the long-term mean, followed by Dow Jones (0.8787) and BET (0.9068). We note that the conditioned volatility of the Romanian indices tends to revert to the mean faster than the other indices included in the study. We proceeded to evaluate the relevance of the GARCH (1.1) through statistical tests on standardized residuals of the model. The GARCH (1.1) is correctly specified if the standardized residuals will no longer show serial correlation, heteroscedasticity or any other linear dependence.

Table 6

Tests for residuals of GARCH (1.1) model

	Standardized residuals			**Squared standardized residuals**			**ARCH-LM (p-val.)**	**Jarque -Bera (p-val.)**
	AC	**PAC**	**Q-stat**	**AC**	**PAC**	**Q-stat**		
DAX	0.051	0.052	28.97 (0.88)	-0.003	-0.007	6.90 (0.99)	0.33 (0.99)	157.9 (0.00)
SPX	0.041	0.039	20.69 (0.41)	-0.014	-0.016	5.97 (0.99)	0.29 (0.99)	505.9 (0.00)
DJI	0.048	0.041	17.09 (0.64)	-0.011	-0.017	10.61 (0.95)	0.53 (0.95)	150.2 (0.00)

	Standardized residuals			Squared standardized residuals			ARCH-LM (p-val.)	Jarque -Bera (p-val.)
	AC	PAC	Q-stat	AC	PAC	Q-stat		
BET	-0.026	-0.026	18.73 (0.53)	0.021	0.023	6.20 (0.99)	0.31 (0.99)	1505.9 (0.00)
BET-BK	-0.015	-0.013	15.18 (0.76)	-0.001	-0.002	8.87 (0.98)	0.42 (0.98)	1665.7 (0.00)
BGTR30	0.066	0.056	37.35 (0.01)	-0.015	-0.021	16.27 (0.7)	0.79 (0.72)	316.3 (0.00)
BUX	-0.019	-0.026	29.64 (0.07)	0.003	0.003	10.90 (0.94)	0.54 (0.94)	207.0 (0.00)
CROBEX	0.042	0.032	30.06 (0.06)	0.029	0.027	13.07 (0.87)	0.62 (0.89)	480.6 (0.00)
FTSE	0.012	0.015	20.18 (0.44)	0.004	0.002	6.05 (0.99)	0.29 (0.99)	91.4 (0.00)
KAX	-0.027	-0.027	15.51 (0.74)	0.007	0.003	12.63 (0.89)	0.55 (0.94)	100.5 (0.00)
OMX30	0.06	0.053	30.37 (0.06)	0.014	0.013	11.63 (0.92)	0.59 (0.92)	105.9 (0.00)
PX	-0.039	-0.035	24.62 (0.21)	-0.023	-0.028	15.21 (0.76)	0.78 (0.73)	273.7 (0.00)
DWIG30	0.009	0.015	23.52 (0.26)	-0.017	-0.022	23.76 (0.25)	1.12 (0.31)	136.0 (0.00)
SMI	0.009	0.009	24.63 (0.21)	0.005	0.006	12.56 (0.89)	0.62 (0.89)	729.0 (0.00)

Source: calculations by the authors

To verify the existence of serial correlations in standardized residuals, we investigated autocorrelation (AC function), partial correlation (PAC function) and applied the Ljung-Q-Box test till the 20-th lag.

We applied the ARCH test (using Lagrange multiplier) to investigate whether we still have ARCH effects in residuals.

The model is appropriate if we notice the lack of these effects. We also applied the Jarque-Bera test to see if the residuals are normally distributed or not. The results of these tests are summarized in Table 6.

The data (presented in Table 6) leads us to conclude that the GARCH (1.1) model is relevant to the analysed financial data series. Simple standardized and squared standardized residuals are not auto-correlated as shown results of AC, PAC, and Q tests.

The ARCH-LM test tells us that there are no ARCH effects in residuals, so the GARCH (1.1) is correctly specified.

The Jarque-Bera test indicates that residuals are not normally distributed, but this feature is often found in the residuals of the appropriate models for the financial time series.

The results of applying the GARCH-M model (1.1) on the same series of financial data are presented in Table 7 of the Annexes. In the table we find the values of the coefficients β, ω, α and β of this model. We started from the assumption that the errors are normally distributed.

The data presented in the table leads us to the following conclusions:

1) The coefficients of the variant equation (ω, α and β) are statistically significant for most time series at the 99% confidence level, with one exception WIG30, whose GARCH coefficient is statistically significant at the 95% confidence level. We have high values for z-statistical and p-low value;

2) Estimated coefficients of the model fulfils the requirement that α + β <1, an essential condition for a mean reverting process. We can conclude that conditional volatilities are mean reverting for all the returns series (except the data tome for WIG30).

3) Unfortunately, the β1 coefficient of the variance term in the mean equation is positive and statistically significant (at the 90% confidence level) only for six series of financial data, respectively SPX, DJIA, FTSE (major indices) and BET, BET- BK and BUX.

Interesting to note, the last three indices are representative indices for the capital markets in Romania and Hungary, neighbouring countries. For the BET index, the results are in line with most of the previous studies and in contradiction with one conducted over the period 1997-2012 showing that the application of the GARCH-M (1.1) model failed to eliminate the ARCH effects of the standardized residual series for daily frequency data.

In conclusion, other GARCH models should be better for modelling and forecasting volatility for the remaining eight times series, for which GARCH-M (1,1) was not the appropriate model. In Table 8 we can see that the conditioned volatility for the BET-BK returns has the fastest mean reverting tendency followed by the Dow Jones and BET returns series.

Table 8

The persistence value in the GARCH-M (111)

Indices	Persistence
DAX	0.985299
SPX	0.911508
DJI	0.881048
BET	0.904979
BET-BK	0.782398
BGTR30	0.950732
BUX	0.934464
CROBEX	0.93522
FTSE	0.935373
KAX	0.979206
OMX30	0.975613
PX	0.954027
WIG30	-0.20512
SMI	0.972802

Source: calculations by the authors

We proceeded to evaluate the relevance of the GARCH-M (1.1) through statistical tests on standardized residuals of the model. GARCH-M (1.1) is correctly specified if the standardized residuals will no longer show serial correlation, heteroscedasticity or any other linear dependence.

To verify the existence of serial correlations in standardized residuals, we investigated autocorrelation (AC function), partial correlation (PAC function) and applied the Ljung-Box test till the 20-th lag. We applied the ARCH test (using Lagrange multiplier) to investigate whether we still have ARCH effects in residuals. The model is appropriate if we notice the lack of these effects. We also applied the Jarque-Bera test to see if the residuals are normally distributed or not. The results are summarized below (Table 9).

Table 9

Tests for residuals of GARCH-M (1.1) model

	Standardized residuals			**Squared standardized residuals**			**ARCH-LM (p-val.)**	**Jarque-Bera (p-val.)**
	AC	**PAC**	**Q-stat**	**AC**	**PAC**	**Q-stat**		
DAX	0.05	0.053	29.02 (0.08)	-0.003	-0.006	6.76 (0.99)	0.33 (0.99)	159.59 (0.00)
SPX	0.045	0.043	19.77 (0.47)	-0.009	-0.011	5.51 (0.99)	0.27 (0.99)	487.92 (0.00)
DJI	0.05	0.043	18.09 (0.58)	-0.006	-0.012	10.31 (0.96)	0.51 (0.96)	156.55 (0.00)
BET	-0.028	-0.028	21.128 (0.39)	0.021	0.022	6.09 (0.99)	0.30 (0.99)	1477.7 (0.00)
BET-BK	-0.017	-0.015	20.09 (0.45)	-0.002	-0.004	8.54 (0.98)	0.40 (0.99)	1691.2 (0.00)
BGTR30	0.066	0.056	37.47 (0.01)	-0.014	-0.02	16.55 (0.68)	0.84 (0.70)	307.84 (0.00)
BUX	-0.022	-0.029	29.07 (0.086)	0.002	0.002	11.10 (0.94)	0.54 (0.94)	215.5 (0.00)
CROBEX	0.041	0.032	29.59 (0.07)	0.028	0.026	12.78 (0.88)	0.61 (0.90)	469.55 (0.00)
FTSE	0.017	0.023	17.77 (0.60)	0.003	0.002	6.05(0.99)	0.29 (0.99)	91.46 (0.00)
KAX	-0.026	-0.026	14.99 (0.77)	0.011	0.006	12.37 (0.90)	0.55 (0.91)	103.89 (0.00)
OMX30	0.062	0.057	29.44 (0.08)	0.014	0.012	10.93 (0.94)	0.56 (0.93)	118,16 (0.00)
PX	-0.04	-0.036	25.03 (0.2)	-0.023	-0.028	15.13 (0.76)	0.77 (0.74)	269,33 (0.00)
WIG30	0.009	0.015	23.52 (0.26)	-0.018	-0.022	23.57 (0.26)	1.11 (0.32)	139.73 (0.00)
SMI	0.011	0.014	22.17 (0.33)	0.004	0.006	12.02 (0.91)	0.59 (0.91)	760,78 (0.00)

Source: calculations by the authors

From the test results presented in Table 9, we conclude that the GARCH-M (1.1) models used to characterize the volatility of financial time series for the fourteen indices are correctly specified.

The results of AC, PAC, and Q statistics test show that there is not statistically significant trace of autocorrelation in standardized residuals. The ARCH-LM test results also show that the model succeeded to eliminate all ARCH effects in residual series.

The Jarque-Bera test indicates that residuals are not normally distributed, but this feature is often found in the residuals of the appropriate models for the financial time series.

In conclusion, the GARCH-M (1,1) model is relevant to our financial data series, the model being able to eliminate the heteroscedasticity and ARCH effects of the daily series of standardized residuals, but the positive correlation between risk and expected return was confirmed for the less than half of the data series. It is recommended to test other models in the GARCH family, maybe asymmetric models, to study the volatility behaviour of these series of financial data in order to identify the most relevant model.

5. Conclusions

In this paper, we used GARCH (1.1) and GARCH-M (101) to characterize volatility on different European and American capital markets. The study included data for fourteen stock indices (from Europe and the US) during March 2013- January 2018, with a daily frequency of the price series. Most of the time series present the characteristics of volatility clustering and heteroscedasticity required to apply the GARCH model.

The GARCH (1.1) model proved to be appropriate for modelling the volatility of returns series, the coefficients of the ARCH and GARCH terms being statistically significant with one exception, the returns series for WIG30, where the GARCH coefficient in the conditional variance equation was negative.

The GARCH-M (1.1) model surprised the positive correlation between assumed risk and future returns for only six of the fourteen sets of financial data. Those were: main American stocks indices (SPX and DJIA), London Stock Exchange index (FTSE), Romanian stock indices (BET and BETBK) and the index of the Hungarian stock exchange (BUX). The results obtained in the case of BET are in line with those of the previous studies, but in contradiction with the study during the 1997-2012, which showed that the modelling of volatility through GARCH-M (1,1) failed to eliminate the effects of ARCH in the residuals series of the model. Conclusions for the BET-BK index are an element of originality for this paper, the index being a relatively new on the BSE and less found in other studies.

From the twelve European time series, only three of them could confirm the hypothesis that the increase in volatility leads to an increase in future returns. The three stock indices belong to neighbouring countries, Romania and Hungary. Perhaps this is a similarity of investors' financial behaviour for a geographic region

After application, both models succeeded to eliminate all traces of autocorrelation and ARCH effects in the standardized residuals series. All residual series continued to be not normally distributed, but this feature was often found in the case of the residuals of the models used to test the financial time series.

The coefficients of the two equations (mean and variance) were statistically significant and showed that conditional volatility tended to revert to the long- term mean, except for the WIG30 index. The coefficient of variance in the mean equation was statistically significant and positive for only six of the fourteen series of data. For these, we found a positive correlation between the risk assumed and the future return demanded by investors on this capital markets.

The persistence of volatility in mature capital markets was lower for US (the markets for which information with a potential negative impact had an insignificant and short-term influence, the strong upward trend being not interrupted by any negative news) and much higher for Germany (0.986) and the United Kingdom (0.93).

We also notice that the persistence of volatility was lower on US markets (in line with our expectations mentioned in the start of the study) compared witch one recorded in European markets. In the last case, volatility remained high, thus showing that on the European capital markets the shocks felt much stronger and their effects persisted for longer periods of time.

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APPENDIX

Table 4

Estimated values for GARCH (1.1) coefficients

	Variance eq.	Coefficient	Std. Error	z-Statistic	P-val.
DAX	ω	187.7588	59.30497	3.165988	0.0015
	α	0.061892	0.010537	5.873726	0.0000
	β	0.924114	0.013744	67.23961	0.0000
SPX	ω	16.24768	2.804355	5.793731	0.0000
	α	0.156303	0.023094	6.768035	0.0000
	β	0.759156	0.029648	25.60606	0.0000
DJI	ω	1916.611	333.5570	5.745978	0.0000
	α	0.184579	0.024221	7.620677	0.0000
	β	0.694186	0.040090	17.31570	0.0000
BET	ω	292.6598	57.73191	5.069290	0.0000
	α	0.147711	0.016914	8.733269	0.0000
	β	0.759187	0.028166	26.95392	0.0000
BET-BK	ω	16.17267	2.796304	5.783586	0.0000
	α	0.214069	0.017646	12.13145	0.0000
	β	0.591175	0.043070	13.72595	0.0000
BGTR30	ω	0.381840	0.073424	5.200501	0.0000
	α	0.144880	0.013796	10.50143	0.0000
	β	0.805987	0.021140	38.12686	0.0000
BUX	ω	3790.264	1062.313	3.567936	0.0004
	α	0.087997	0.015591	5.643895	0.0000
	β	0.850801	0.028489	29.86414	0.0000
CROBEX	ω	6.165254	1.214226	5.077518	0.0000
	α	0.076169	0.012519	6.084371	0.0000
	β	0.856086	0.021720	39.41454	0.0000
FTSE	ω	202.1501	40.81481	4.952861	0.0000
	α	0.122284	0.018530	6.599302	0.0000
	β	0.809663	0.026315	30.76857	0.0000
KAX	ω	2.032260	0.599750	3.388512	0.0007
	α	0.110842	0.014631	7.575998	0.0000
	β	0.872949	0.01646	53.00928	0.0000
OMX30	ω	5.311251	1.649986	3.218968	0.0013
	α	0.101532	0.016402	6.190254	0.0000
	β	0.874369	0.022308	39.19476	0.0000
PX	ω	3.297118	0.731536	4.507117	0.0000
	α	0.098451	0.012822	7.678009	0.0000
	β	0.855385	0.018564	46.07786	0.0000
WIG30	ω	727.1751	75.98581	9.569880	0.0000
	α	0.110934	0.022586	4.911624	0.0000
	β	-0.299641	0.101827	-2.942658	0.0033
SMI	ω	199.4564	46.89080	4.253635	0.0000
	α	0.130998	0.015702	8.342855	0.0000
	β	0.842521	0.019915	42.30590	0.0000

Source: calculations by the authors

Table 7

Estimated values for GARCH-M (1.1) coefficients

	Variance eq.	Coefficient	Std. Error	z-Statistic	P-val.
DAX	β_1	-0.009323	0.105018	-0.088773	0.9293
	ω	197.1702	61.00719	3.231917	0.0012
	α	0.064205	0.010842	5.922025	0.0000
	β	0.921094	0.014066	65.48344	0.0000
SPX	β_1	0.400867	0.120986	3.313319	0.0009
	ω	17.03805	2.975056	5.726967	0.0000
	α	0.167453	0.025904	6.464388	0.0000
	β	0.744055	0.033213	22.40283	0.0000
DJI	β_1	0.266370	0.125865	2.116309	0.0343
	ω	1880.606	331.3855	5.674982	0.0000
	α	0.184596	0.024638	7.492196	0.0000
	β	0.696452	0.040368	17.25259	0.0000
BET	β_1	0.293213	0.160875	1.822612	0.0684
	ω	297.6167	59.85748	4.972089	0.0000
	α	0.150351	0.016988	8.850520	0.0000
	β	0.754628	0.029117	25.91668	0.0000
BET-BK	β_1	0.309912	0.180409	1.717828	0.0858
	ω	17.91310	2.929159	6.115442	0.0000
	α	0.224950	0.018297	12.29438	0.0000
	β	0.557448	0.044995	12.38924	0.0000
BGTR30	β_1	-0.080370	0.110019	-0.730503	0.4651
	ω	0.381006	0.073885	5.156764	0.0000
	α	0.144905	0.014039	10.32130	0.0000
	β	0.805827	0.021154	38.09403	0.0000
BUX	β_1	0.300091	0.168632	1.779562	0.0751
	ω	4039.936	1102.252	3.665166	0.0002
	α	0.091377	0.016521	5.531090	0.0000
	β	0.843087	0.029753	28.33599	0.0000
CROBEX	β_1	-0.032104	0.167473	-0.191697	0.8480
	ω	5.927939	1.207571	4.908977	0.0000
	α	0.074998	0.012345	6.074956	0.0000
	β	0.860222	0.021567	39.88514	0.0000
	β_1	0.347581	0.127428	2.727665	0.0064
FTSE	ω	189.9959	40.43424	4.698886	0.0000
	α	0.116626	0.018024	6.470758	0.0000
	β	0.818747	0.026498	30.89884	0.0000
KAX	β_1	0.055858	0.096916	0.576358	0.5644
	ω	2.582926	0.766458	3.369949	0.0008
	α	0.127024	0.017182	7.392875	0.0000
	β	0.852182	0.019947	42.72267	0.0000
OMX30	β_1	0.124306	0.102908	1.207931	0.2271
	ω	5.383833	1.668031	3.227657	0.0012
	α	0.103449	0.017404	5.943945	0.0000
	β	0.872164	0.022760	38.32003	0.0000
PX	β_1	-0.073602	0.138742	-0.530497	0.5958
	ω	3.308343	0.745860	4.435606	0.0000
	α	0.099741	0.012972	7.688670	0.0000
	β	0.854286	0.018728	45.61651	0.0000

	Variance eq.	Coefficient	Std. Error	z-Statistic	P-val.
WIG30	$\beta1$	0.030778	0.270078	0.113958	0.9093
	ω	737.2732	100.5816	7.330102	0.0000
	α	0.106266	0.022565	4.709402	0.0000
	β	-0.311387	0.150800	-2.064893	0.0389
SMI	β_1	0.132316	0.111578	1.185870	0.2357
	ω	204.6108	48.02399	4.260595	0.0000
	α	0.133025	0.015912	8.360233	0.0000
	β	0.839777	0.020149	41.67866	0.0000

Source: calculations by the authors

Figure 1

Graphs of stock indices

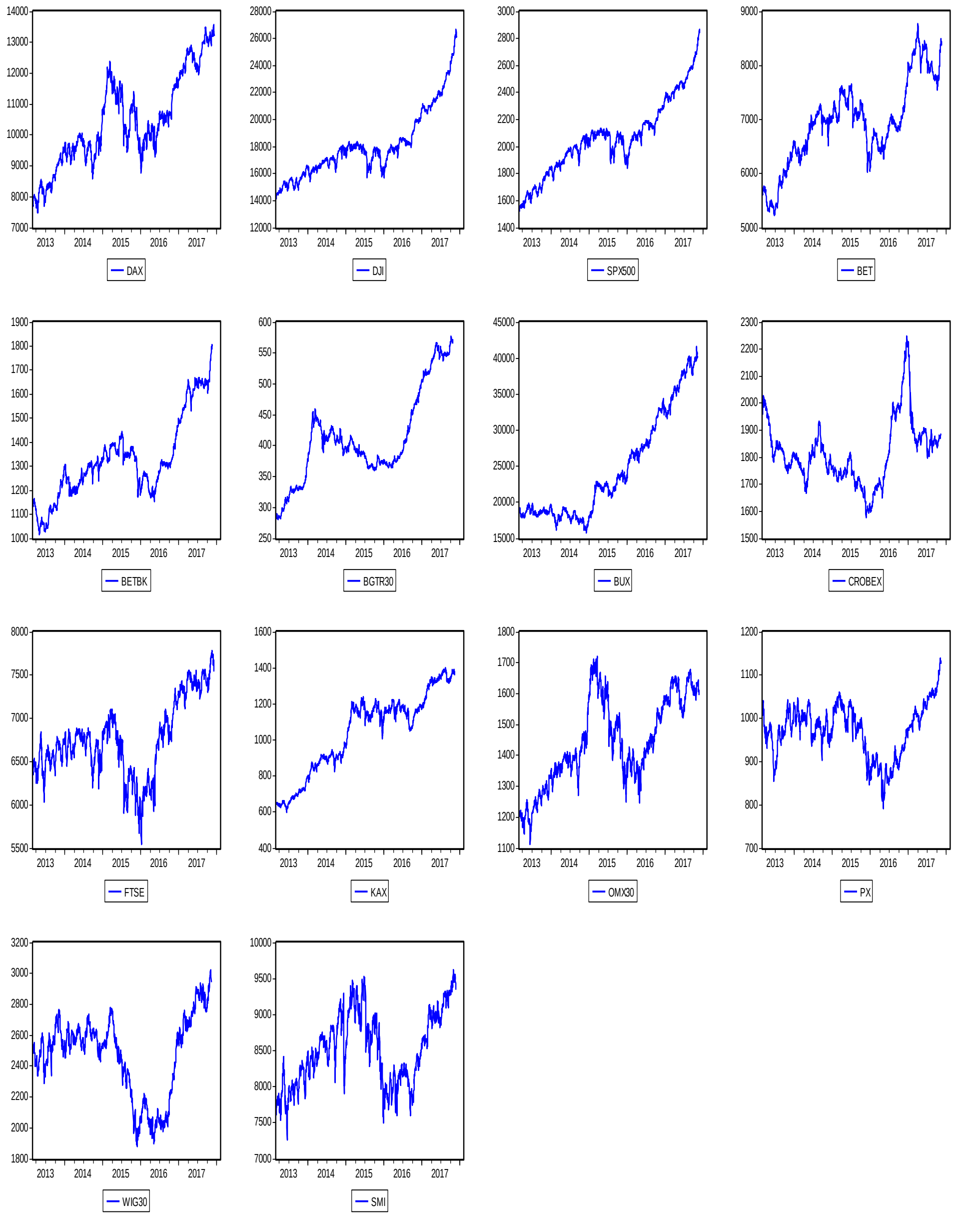
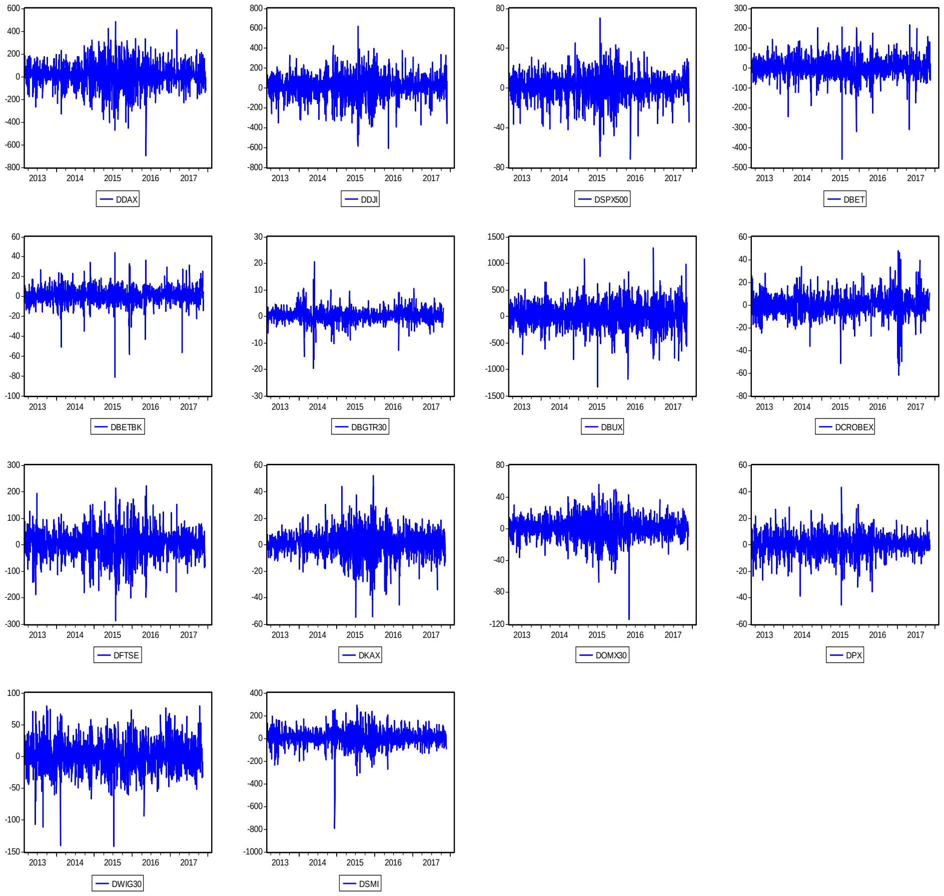


Figure 2

Series of logarithmic returns for all stock indices



Financial Studies

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