CAN BITCOIN BE A STABLE INVESTMENT?

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Abstract

This study aims to analyze the volatility structure of Bitcoin returns, which became a popular investment after 2009. The Fractal Market Hypothesis (FMH) is chosen as the instrument to investigate the issue. By testing this hypothesis, the sudden price fluctuations in Bitcoin returns were tried to be determined. Daily closing price of Bitcoin between 04/2013-01/2019 were obtained from coinmarketcap. The fractal nature of Bitcoin market is tested with R/S, DFA, Periodogram and GPH models. The Hurst exponents show that FMH is valid in the Bitcoin market. Additionally, the effect of financial bubble formation and structural breaks on fractality is investigated through the ARFIMA-FIGARCH and ARFIMA-HYGARCH models. We observe that financial bubbles and regime changes increase the fractal structure (long memory) in the Bitcoin market.

Keywords: Fractal Market Hypothesis, Hurst Exponent, Financial Bubbles, FIGARCH, HYGARCH

JEL Classification: C58, G14

1. Introduction

Efficient Market Hypothesis (EMH) is based on the assumption that all useful information in financial markets is reflected in prices quickly and precisely. EMH emphasizes that future investment prices cannot be predicted, investors cannot obtain abnormal returns and asset prices exhibit random walk behaviour. However, the hypothesis is extensively criticized due to some innovation especially in positive sciences. In particular, Hurst (1951) showed that time series exhibited a biased random process or fractional Brownian motion contrary to pure random walk behaviour (Peters, 1989), which caused the

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discussion of the Efficient Market Hypothesis based on Fama (1970). In addition to the fact that financial asset series do not exhibit random walk behaviour, EMH is also severely criticized in terms of the lack of symmetrical knowledge of the market players and the lack of the normal distribution of asset returns (Morali & Uyar, 2018: 2204). Common assumptions about financial asset returns should be listed as follows. Financial asset returns: a) heavy-tailed according to normal distribution b) autocorrelation and partial autocorrelation functions that do not rapidly approach to 0 c) time series with non-periodic cycles. For this reason, Hurst (1951) and Mandelbrot (1972) emphasized that the financial time series exhibited long memory behaviour and that the events emerging in the past can be small pieces that form the picture of the today and today's useful information may be small pieces that will form the picture of the future.

Fractal can be defined as a repeating pattern of details and structure. The Fractal Market Hypothesis, introduced by Peters et al. (1994), was developed as a result of the understanding of self-similarity and long memory characteristics in financial time series. It contributes more to the understanding of capital markets with excessive volatility, discontinuity and non-periodic character (Rachev et al., 1999: 24). Although fractality is expressed as a geometric concept, it expresses the self-similar principle for financial assets. But, the concept of self-similarity statically indicates that the mean and standard deviation of any part of the fractal is proportional to the mean and standard deviation of the whole entity (Erdoğan, 2017a: 51).

The self-similarity feature of financial time series can be explained by long memory behaviour. Long memory series exhibit nonperiodic long cycles or permanent dependence in observations that are distant from each other over time. Short-term dependent time series includes standard autoregressive moving average and Markov operations and reveals whether the observations show little statistical dependence from each other (Mulligan, 2000: 33).

Long memory and structural breaks are two important factors in modelling financial time series, and they contribute to the elimination of losses caused by excessive volatility affecting asset diversification, financial asset prices and market expectations (Mensi et al. 2019). The fact that financial assets do not exhibit long memory characteristics can be interpreted as the events of the past do not affect today's prices.

This study aims to investigate the fractal nature of BTC returns. Bitcoin was introduced to the financial world through Nakamoto (2008) as an electronic payment system with crypto security systems. It attracted a lot of attention of investors especially during 2009 and has exhibited a rather unstable investment instrument for the last few years. The daily trading volume of BTC returns in the cryptocurrency market is approximately 35% of the total trading volume. Although the sudden and high price changes in the cryptocurrency markets provide opportunities for serious speculative gains, they also cause many highlevel risks. Particularly, empirical evidence on the persistence level of information shocks can give an idea about whether crypto financial instruments are reliable investment instruments for investors. Within the cryptocurrency market, BTC has a large share of the total transaction volume including more than 2000 cryptocurrencies. Especially, the limited supply leads to higher volatility in return.

Different from the previous studies, the unique aspect of our study is that the BTC market fractality will be examined separately in the context of both Hurst exponent and d parameter. Furthermore, Fractal Market Hypothesis will be tested by considering the structural breaks, financial bubble formations and dual long memory characteristics. There are only a few studies in the literature which test the Fractal Market Hypothesis from different dimensions in the crypto money market by considering the information efficiency of the BTC market.

The rest of the paper is organized as follows: Section 2 presents the literature review about efficiency, volatility structure, fractality and long memory in the BTC market. Section 3 defines the data set and methodology. Empirical findings will be presented in Section 4. Finally, Section 5 concludes.

2. Literature review

While the first studies on Bitcoin focused on issues such as "security" or "the legality" of cryptocurrencies, studies emerging after 2013 have emphasized the financial aspects of the cryptocurrencies (Kristoufek, 2018: 257). As Bitcoin becomes popular in portfolio diversification, many studies on "market efficiency", "long memory", "price discovery", and "hedging capabilities" of BTC have been conducted in the literature.

Urquhart (2016, 2018), Bariviera (2017), and Kristoufek (2018) examined whether the crypto money market was efficient by means of BTC prices for different periods. The common result of the analysis

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conducted with different models such as Hurst R/S and the run test was is that BTC contradicted the Efficient Market Hypothesis. Similarly, Lahmiri et al. (2018), Al-Yahyaee et al. (2018), Lahmiri and Bekiros (2018), Mensi et al. (2019), Kayacan and Anavatan (2018) and Erdoğan (2018) examined the multi-fractal structure and dual long memory properties in BTC markets via various parametric and semi-parametric models. Under the assumptions of different forms of error distribution, long memory and fractality were tried to be determined with different model variations such as Hurst R/S, MF-DFA (Multifractal De-trended Fluctuation Analysis), BDS (Brock, Dechert and Scheinkman), FIGARCH, FIAPARCH, and HYGARCH. It is shown that the BTC market was a multi-fractal market with a long memory character.

In their study, Katsiampa (2017) and Dyhrberg (2016a) revealed the volatility characteristics of the BTC market. Dyhrberg (2016a) tested the GARCH and EGARCH models in a study aiming to demonstrate the capabilities of BTC as a financial asset. According to the results of the GARCH model, BTC had some similarities in terms of gold and dollar hedging capabilities as a portfolio diversification tool. As a result of the analysis of the EGARCH model, it was found that BTC offered opportunities for investors expecting negative shocks in the market. Katsiampa (2017) showed that the AR-CGARCH was the best predictor of the BTC volatility among many conditional variance estimators for determining BTC volatility structure. In another study, Dyhrberg (2016b) found that BTC offered hedge opportunities compared to the stock.

Apart from the aforementioned issues on cryptocurrencies, there exist some studies on the existence of structural breaks and price bubbles. Corbet et al. (2018) obtained some evidence of price bubbles in both currencies via SADF and GSADF tests in their study aiming to identify the price bubble formations in Bitcoin and Ethereum by following the principles proposed by Phillips et al. (2011, 2015)

Thies and Molnar (2018) tried to detect structural breaks in BTC returns and volatility through Bayesian models and identified different positive average return regimes and one negative regime as indicators of structural breaks in both return and volatility.

Regime differences such as structural breaks and price bubbles in volatility modelling of financial assets should be considered to avoid inaccurate volatility predictions. Thus, the regime changes (structural breaks) and financial bubbles in volatility models to be used in testing the Fractal Market Hypothesis in the BTC market have been included in this study.

3. The contribution of Post-Keynesian economics

We obtained daily data for BTC from coinmarketcap for the period from 28/04/2013 to 25/01/2019. We compute the log returns via $ln(P_t/P_{t-1})$.

It is worth emphasizing that the fractality observed in the financial time series leads to long memory and therefore information shocks that reach the market will not be geometrically reflected in the prices in the market. The fractality in the structure of a long-term financial asset indicates that future price formations can be predicted by means of past observations, which provides evidence for the Fractal Market Hypothesis contrary to the Efficient Markets Hypothesis.

The Hurst Exponent (self-similarity coefficient) described by Hurst (1951) is explained by the following asymptotic relationship.

$$E\left[\frac{R(n)}{S(n)}\right] \sim \alpha * n^{H}, n \to \infty$$
(1)

In equation, $R_{(n)}/S_{(n)}$ indicates the rescaled range, α is the constant term, $R_{(n)}$ means the difference between the largest and smallest value of the series and $S_{(n)}$ refers to the standard deviation of the series. It is possible to calculate the Hurst exponent in Equation 1 more easily by logarithmic transformation. When equation 1 is expressed in logarithmic form, the Hurst exponent represents the curve of the line (Brooks (1995), Šiljak and Şeker (2014), Beran (1994), Aygören (2008), Morali and Uyar (2018)).

$$log(\frac{R}{S})_n = log_{(\alpha)} + H^* log_{(n)}$$
(2)

The motivation for using the Hurst exponent stems from the spreading characteristics of the time series integrated with the past. The fact that the Hurst exponent, which can take values between 0 and 1, and is used to interpret the rate of decaying of the autocorrelation function of the time series, is less than 0.5 indicates that the series exhibits a short memory feature. That is, the autocorrelation function decays rapidly.

If the Hurst exponent lies in the interval (0.5,1), then it means that the series exhibits long memory and reluctant to approach to the average. In the long-memory series, the fact that small pieces of the past contribute to inferring future asset price movements will provide evidence of the validity of the Fractal Market Hypothesis (Hurst (1951), (Aygören (2008), (Mulligan (2000)).

There are many methods to estimate the self-similarity parameter H or the intensity of long-range dependence in a time series (Taqqu et al., 1995: 785). In the application part of the study, Detrended Fluctuation Analysis (DFA), Geweke and Porter-Hudak (1983) (GPH), Taqqu et al. (1995) Periodogram, Modified Periodogram (M-Per) and R / S methods discussed by Mandelbrot (1972) and Mandelbrot and Taqqu (1979) were used to calculate H and d parameters.

In De-trended Fluctuation Analysis (DFA) developed by Peng et al. (1994), the time series is de-trended in 3 steps, and the Hurst exponent is calculated. First, the average of the series is obtained so that each element of the time series can be distinguished from the average. The y series, which represents the sum obtained from the differences from the average, is divided into m equal parts, resulting in the y_m series. The OLS is estimated for each local part divided into m parts. Finally, by subtracting the trend from each local series, the integrated time series becomes de-trended ((Weron, 2007: 52), (Peng et al. 1994), (Erdogan, 2017b: 557)). DFA analysis is also more resistant and less sensitive to the possibility of the series leaving the stationary conditions (Bariviera, 2017).

Another method used to calculate the Hurst exponent is the semi-parametric GPH method proposed by Geweke and Porter-Hudak (1983) and based on the calculation of the fractional integration parameter (d). Hurst exponent can be shown as H = d + 0.5 (Weron, 2007: 53). Therefore, GPH, a gaussian method, can be used to calculate the Hurst exponent. Periodogram analysis is used to reveal the dominant period or periods in the time series (Erdoğan, 2017a: 52).

Fractional integration parameter (ξ , d), which is calculated via parametric tests, can also be used to test long memory. Fractality can be tested in both BTC returns and yield volatility using the ARFIMA (p, ξ ,q) model developed by Granger and Joyeux (1980), and Hosking (1981), the FIGARCH (p,d,q) method proposed by Baillie et al. (1996) and the Hyperbolic GARCH (HYGARCH) model developed by Davidson (2004).

The different rate of decaying in the autocorrelation functions of the time series prevents the stationarity levels of the series from being represented as absolute numbers such as I[0] and I[1]. The possible fractal structure in the return and volatility of financial assets leads to a differentiation in the rate of decaying of information shocks affecting financial assets, thus indicating that the long-term conditional returns and conditional volatilities of financial assets with long memory characteristics are predictable. This result implies the emergence of evidence supporting the fractal market hypothesis for the financial assets contrary to the efficient market in the weak form.

The fractional integration (d) level of conditional returns of financial assets can be calculated using the ARFIMA model introduced by Granger and Joyeux (1980) and Hosking (1981).

$$\Psi(L)(1-L)^{\xi}(y_t - \mu) = \theta(L)\varepsilon_t$$
(3)

$$\varepsilon_t = z_t \sigma_t \tag{4}$$

$$z_t \sim ST(0,1,v) \tag{5}$$

In equation 3, ξ refers to the long memory parameter on the conditional mean, and L represents the lag operator. According to Hosking (1981) in the autoregressive fractionally integrated moving average (ARFIMA) model,

a) $-0.5 < \xi < 0.5$ values are stationary and invertible

b) ξ = 0 series is stationary (short memory)

c) ξ = 1 a unit root process

d) $0 < \xi < 0.5$ series is with long memory (positive dependent with distant observations)

e) $-0.5 < \xi < 0$ anti-persistent long memory (negative dependent with distant observations) (Mensi et al. 2019)

The FIGARCH model developed by Baillie et al (1996) enables the calculation of fractional integration (d) parameter in the return volatilities of financial assets by considering the possibility that the effects of information sets affecting financial assets will decrease at a hyperbolic rate in shaping the conditional variance of the future. The standard FIGARCH (p, d, q) model is given in equation 6.

$$\sigma_t^2 = \omega [1 - \beta(L)]^{-1} + [1 - [1 - \beta(L)]^{-1} \phi(L) (1 - L)^d] \varepsilon_t^2$$
(6)

 ω, β, ϕ , and *d* in the equation represent the fixed term, GARCH term, ARCH term and long memory parameter in conditional variance, respectively. $(1 - L)^d$ refers to the fractional integration operator in the conditional variance equation. All values between 0 <d <1 indicate the presence of long memory (fractal) in the volatility of the series.

The fact that the fractional integration parameter (ξ , (d)) equals 0 in both the return and volatility series indicates short memory, emphasizing that the effect of information shocks on the financial asset disappears at the geometric speed. As ξ and / or d moves away from 0, the series should be interpreted as exhibiting long-term positive dependence.

 α is the stationarity criterion in the HYGARCH model developed by Davidson (2004) as a generalized version of FIGARCH and given by equation 7. In the HYGARCH model, when $\alpha < 0$, the covariance of the process is stationary, i.e. autocorrelation roots decrease. The value $\phi(L)/\beta(L)$ represents the rate of decaying of the shock.

$$\sigma_t^2 = \omega [1 - \beta(L)]^{-1} + \{1 - [1 - \beta(L)]^{-1} \phi(L)(1 + \alpha [(1 - L)^d - 1])\} \varepsilon_t^2 \quad (7)$$

$$\alpha \ge 0, d \ge 0$$

4. Empirical results

When the graph regarding BTC returns is analysed, it can be seen that there are volatility clusters and regime changes as an indicator of structural breaks became more frequent after 2017 especially with the increase in popularity.

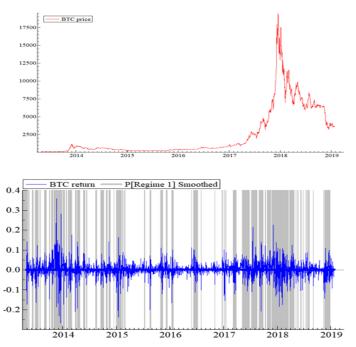
From the BTC returns graph we observe that there are volatility clusters and regime changes as indicators of structural breaks, and they become more frequent after 2017 especially with the increase in popularity.

Figure 1 shows Bitcoin (BTC) closing prices and logarithmic returns. In the return graph, regime changes and volatility clusters can easily be seen with the help of shaded areas. The MS-DR test was used to detect deviations from the mean yield, and stable and volatile regimes were separated.

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Figure 1

BCT graphs of the price and logarithmic return series, the shaded area in the return graph represents the regime changes obtained from the MS-DR



Source: Prepared by the author

Table 1 presents descriptive statistics. The stationarity of BTC logarithmic returns whose unit root test results are presented can be understood via the ADF and KPSS unit root tests, whereas the normality can be seen with skewness, kurtosis and jarque bera tests. In addition, it was found that the errors and squared error contain autocorrelation (Q and Q²) and also have conditional variance (ARCH_LM).

Table 1

Statistical Properties of BTC Return

	BTC	
	Log. Return	
Mean	0.001567	
Maximum	0.35745	
Minimum	-0.2662	

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Std. Deviation	0.043703	
Skewness	-0.18635	
Kurtosis	7.8093	
Jarque Bera	5,345.9***	
ARĈH_LM	112.22***	
Q_20	56.8411***	
Q^{2}_{2}	744.196***	
ADF	-26.6232***	
KPSS	0.155231	

*Notes: *** denotes significance at the 1% level.*

Source: Prepared by the author

The Hurst Exponent is calculated by using the methods mentioned in the method section for testing the Fractal Market Hypothesis in BTC returns and volatility (quadratic returns) and the results are presented in Table 2. In order to make robust estimation Hurst exponent was calculated by using more than one method and it was found to be in the range of 0.5302-0.6565 for logarithmic return series and 0.6876-0.9837 for the squared return series representing volatility. The calculation of the Hurst exponent in the range of 0.5-1 proves that the FPH hypothesis is valid in the BTC market and there is a long-term shock persistence.

Table 2

Hurst Exponents of BTC Return and Volatility Series

	BTC	BTC
	Log. Return	Squared Return (volatility)
DFA	0.5848	0.8051
GPH	0.6142	0.9837
Periodogram	0.6267	0.7244
M-Periodogram	0.5302	0.6876
R/S	0.6565	0.7506

Table 3

Estimate Results of ARFIMA (1, ξ , 1) -GARCH (1,1) Type Model of BTC Logarithmic Returns

	FIGAR	CH(1,d,1)-st	HYGARCI	H(1,d,1)-st
	I	втс		С
μ	0.002232**	(0.0010531)	0.002162**	(0.00098118)
AR(1)	0.434492***	(0.074457)	0.423411***	(0.067630)
ξ	0.152523**	(0.067676)	0.155644**	(0.062515)
MA(1)	-0.618074***	(0.088968)	-0.616093***	(0.081341)
ω	0.154381	(0.13648)	-0.205306	(0.28680)
d_figarch	0.721058***	(0.096658)	0.587950***	(0.097924)

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ARCH(1)	0.223491***	(0.063448)	0.300791***	(0.11606)
GARCH(1)	0.734919***	(0.072242)	0.713918***	(0.084751)
Log(a)			0.427297**	(0.18671)
Student_df	3.297112***	(0.14599)	2.435179***	(0.15331)
AIC	-4.026118		-4.039	353
BIC	-4.001895		-4.012	439
HQ	-4.017245		-4.029	495
Log-Likelihood	4234.41		4249	0.3
$Q^{2}(20)$	17.5326		17.5863	
ARCH(10)	1.2567		1.33	70

Note: () represents Standard Errors. ** and *** indicate the significance level of 5% and 1%.

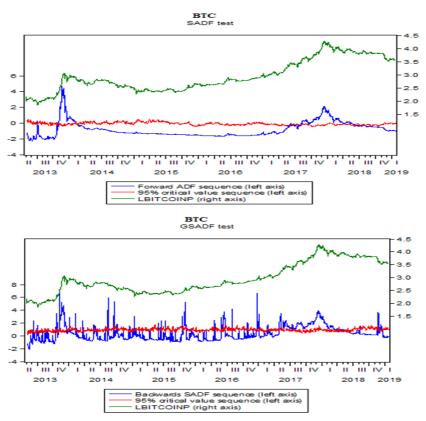
Source: Prepared by the author

According to the results, both long memory (ξ) on return and long memory (d) parameters on volatility were found to be statistically significant. It is understood from the Q² and ARCH test results that the error squares obtained from the FIGARCH and HYGARCH model results do not contain autocorrelation and do not have varying variance. In the HYGARCH model Log(α) <1 was found. This result shows that autocorrelation roots decrease and the HYGARCH model is more stable than the GARCH and IGARCH model. When the information criteria and Log-likelihood values of the models are examined, it is seen that the ARFIMA-HYGARCH model is more successful in modelling volatility in BTC market. Similar to the Hurst exponent results presented in Table 2, the ARFIMA-FIGARCH and ARFIMA-HYGARCH results provide support for the existence of Long Memory and FPH in the BTC market.

To determine the possible effects of regime changes on fractality and long memory behaviour, SADF and GSADF tests (Phillips et al. (2011, 2015)), were used. Additionally, financial bubble formation and structural breaks were analysed by the Markov Switching Dynamic Regression model.

SADF and GSADF tests, which are based on recursive regression, try to identify sudden price increases i.e. financial bubbles that occur differently from the specific behaviour of financial assets (Phillips et al., 2011). These tests are a right-tailed variant of the ADF unit root test. In the SADF test, recursive regression estimations are used to determine the burst behaviour in which the bubbles occur in the financial asset and then the point of collapse. The GSADF test (Phillips et al. 2015) tries to overcome the lack of SADF test to detect

single financial bubble formation. It is used to detect multiple bursts and crashes. Figure 2 shows both SADF and GSADF test results. The test statistics reached after 1000 simulations in which the initial window size was set to 10 were greater than the critical value. Therefore, the null hypothesis "there is no financial bubble formation in BTC prices" was rejected.



SADF and GSADF Test Results of BTC Prices

Figure 2

Source: Prepared by the author

ARFIMA-FIGARCH and ARFIMA-HYGARCH models were reestimated to determine whether both financial bubble formation and high volatility regime changes had an impact on the fractality of the BTC market and the results are presented in Table 4.

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Both tests show that the dummy variables representing the high volatility regime and the financial bubbles are statistically significant in terms of representing structural breaks. According to the dummy model results, a decrease in the long memory values is easily understood from both ξ and d parameters compared to the dual long memory parameters in the dummy-free models. The results showed that structural breaks ($D_{Structural breaks}$) and financial bubble ($D_{bubbles}$) (formations are factors that increase the fractal structure. In the dummy model, long memory parameters are either reduced or meaningless. It is seen in Q² and ARCH test results that the error squares obtained from the dummy model results do not contain autocorrelation and have constant variance.

Table 4

Estimate Results of ARFIMA (1, ξ, ⁴	1) -GARCH ([^]	1,1) Type	Model of
BTC Logarithmic Returns (with Dummy	Variable)

	FIGARCH(1,d,1)-st		HYGARCH(1,d,1)-st		
	ВТС		BTC		
μ	0.002145***	(0.00065456)	0.001843***	(0.00067009)	
AR (1)	0.457948***	(0.092007)	0.449539***	(0.087918)	
ξ	0.093959**	(0.047272)	0.101723	(0.052984)	
MA(1)	-0.596655***	(0.10195)	- 0.592509***	(0.099999)	
ω	23.460138***	(0.30994)	8.451175***	(0.17078)	
d_figarch	0.042986***	(0.0068467)	0.342306***	(0.10810)	
Dstructural break	-0.002380***	(0.0000022)	- 0.000863***	(0.000000092	
Dbubbles	0.000535***	(0.00010770)	0.000681***	(0.00020669)	
ARCH(1)	0.026608	(0.044457)	0.282986***	(0.057400)	
GARCH(1)	0.045670	(0.040377)	0.415716***	(0.068603)	
Log(a)			- 0.693945***	(0.081248)	
Student_df	6.297169***	(0.15246)	5.324590***	(0.46718)	
AIC	-4.29	-4.292303		4.225124	
BIC	-4.262697		-4.192827		
HQ	-4.281459		-4.213294		
Log-Likelihood	4515.772		4446.267		
Q ² (20)	36.7405**		14.7055		
ARCH(10)	1.5546		0.60263		

Note: () represents Standard Errors. ** and *** indicate the significance level of 5% and 1%.

Source: Prepared by the author

5. Conclusions

Bitcoin, which has recently been in the foreground in terms of transaction volume in the cryptocurrencies, was introduced to the market in the form of the crypto payment system in 2008. However, it has attracted the attention of investors as speculative earning alternatives in which price fluctuations have been observed. The rate of increase in prices in 2017 caused some concerns about possible financial bubbles in the Bitcoin market. Whether the Bitcoin market is a reliable investment tool for rational investors has become an important topic to be investigated by the researchers. The question "Should the dollar remain as reserve money for countries, especially because of the seigniorage income?" has attracted many interest recently, and the information of the preliminary studies regarding the question of whether crypto payment systems can be a new reserve instrument for countries has circulated around.

The Fractal Market Hypothesis, introduced by Peters et al. (1994) into the financial literature as a counter-thesis of the Effective Market Hypothesis, tries to convey the self-similarity concept that the current returns of financial assets will carry parts of the past. To test the validity of the Fractal Market Hypothesis in the BTC market, Hurst exponents have been calculated using multiple models. The results showed that Hurst exponent in the BTC market is in the range of 0.53-0.65 in the return series and 0.68-0.98 in the squared return series included in the analysis to represent volatility. This result can be interpreted as the validity of the Fractal Market Hypothesis in the BTC market.

Fractality is also an issue referred to long memory. Thus, the dual long memory behavior in both the return and volatility in the BTC market has been re-analyzed with ARFIMA-FIGARCH and ARFIMA-HYGARCH models. We conclude that there is dual long memory in both return and volatility, useful information shocks reaching the BTC market are reflected in prices at a hyperbolic rate, and the shock has a long memory effect. This result is consistent with fractality. The SADF and GSADF tests (Phillips et al. (2011 and 2015)), were conducted to determine whether possible financial bubble formations and regime changes affect the Fractal structure (Long Memory) in the BTC market, and it was determined that there were financial bubble formations in the BTC market. In addition, periods with high volatility were determined by the MS-Dynamic Regression method.

The ARFIMA-FIGARCH and ARFIMA-HYGARCH models were re-estimated by using dummy variables that represent financial bubble and structural breaks, and it was concluded that dummy variables were significant and increased Fractality (long memory) in the market. The results show that the Bitcoin market is ineffective and that financial bubble formations and regime changes are one of the most important sources of resistance in reaching information efficiency. The results of this study are similar to the study of Corbet (2018) investigating the formation of financial bubbles in cryptocurrencies, and the studies of Kristoufek (2018), Bariviera (2017), Lahmiri et al. (2018), Mensi et al. (2019), Urguhart (2016), Lahmiri and Bekiros (2018), and Al-Yahyaee et al. (2018) determining that the BTC is an inefficient market. Bitcoin's ability to become a stable investment instrument is closely related to the efforts to decrease volatility in the market. The current study can be extended by applying the methodology described in this paper to the other cryptocurrencies which are representative of the market.

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