CAUSAL RELATIONSHIP BETWEEN BITCOIN PRICE VOLATILITY AND TRADING VOLUME: ROLLING WINDOW APPROACH

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Abstract

This study investigates the causal relationship between price volatility and trading volume for bitcoin which is the first cryptocurrency. Data are daily and cover the period starting from December 27, 2013 to March 3, 2019. Price volatility series was produced by using EGARCH model. The Toda-Yamamoto causality test was applied under rolling window approach. According to the Granger causality test, there is a strong causal relationship running from the trading volume to the price volatility. There also exists a causality running from price volatility to volume. But this causality is not statistically strong. At the same time, a positive and significant contemporaneous correlation was found between the two variables. Both findings support the sequential information arrival hypothesis for the bitcoin market.

Keywords: sequential information arrival hypothesis, Toda-Yamamoto causality, cryptocurrency

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1. Introduction

In the finance literature, the causal relationships between the price volatility and trading volume of any asset has long been the subject of discussion. There are two fundamental hypotheses on the dynamic relations between these two variables. One of them is the

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mixture of distribution hypothesis developed by Clark (1973), Epps and Epps (1976), Harris (1986) and Anderson (1996). The mixture of distribution hypothesis indicates the existence of a positive contemporaneous correlation between asset prices and trading volume. The variance of the price change in a single transaction depends on the volume of this transaction. Therefore, the relationship between price volatility and trading volume is based on a fundamental variable called the rate of information flow into the market. Price and trading volume change at the same time. According to this hypothesis, there is no causal relationship between two variables. The other hypothesis on the subject of the relationship between price volatility and trading volume is the sequential information arrival hypothesis. This hypothesis was developed by Copeland (1976) and Jennings et al. (1981), and Smirlock and Starks (1985). The sequential information arrival hypothesis assumes that new information is sequential in terms of the buyers and sellers in the asset market. In the beginning, buyers and sellers are in equilibrium because they have the same set of information. As new information arrives, buyers and sellers may revise their expectations again. However, buyers and sellers cannot receive information signals simultaneously. When all market participants receive new incoming information and according to it they revise their expectations, then the final equilibrium takes place. In this hypothesis, the sequential response to information suggests that there must be a bidirectional causal relationship between price volatility and trading volume.

In the relevant empirical literature, there are numerous studies which test these hypotheses with different econometric approaches. Almost all of the current empirical studies in the literature have investigated the relationship between the two variables for the stock, bond and equity markets. The results are generally that there is a bi-directional causality between two variables. Hiemstra and Jones (1994), Kim et al. (2005), Chen and Wu (2009), Mahajan and Singh (2009), Chiang et al. (2010) and Chan et al. (2018) are some of the studies supporting the sequential information arrival hypothesis. The question to be answered at this stage is whether the findings obtained for the traditional asset markets are also valid to cryptocurriencies with both monetary and asset functions. In other words, does the bidirectional relationship between price volatility and trading volume apply to cryptocurriencies?

As known, bitcoin, the first of the cryptocurrency, was developed in 2009 by a person or group known as Satoshi Nakamoto. Bitcoin, which was circulated as virtual money, was known by very few people in the early days of its emergence, but it has started to be widely traded in the money and financial environment that have put the current international money system in serious danger for the last two years. There is general evidence that the existence, direction and severity of the causal relationships between price and volume in monetary and financial markets depend on the trading volume. For this reason, it is expected that the possible causal relationship between bitcoin price volatility and the trading volume can be strengthened with the increasing trading volume.

In order to answer the above question, the present study examines the dynamic progress of the possible causal relationships between the daily price volatility and the daily trading volume of bitcoin by using the approach of rolling window causality test developed by Hill (2007).

2. Data and econometric method

In the study, daily data were used for the period December 27, 2013 – March 3, 2019. Data on the daily closing price and trading volume of bitcoin are available from coinmarketcap.com. Since the cryptocurrency market is active every day of the week, the data set used in this study covers every day of the year. The natural logarithmic transformations of price and trading volume of bitcoin were taken before the causality test. Then, the volatility series of the bitcoin price was produced by appropriate autoregressive conditional heteroskedasticity (ARCH) model. Finally, the causal relationship between price volatility and trading volume was determined by using the rolling window causality approach.

In traditional econometric models, the variance of the error term is assumed to be constant. However, even though the unconditional variance of the error terms in the time series is constant, the conditional variance may not be constant. It is difficult to provide the assumption that the conditional variance of the error term is constant, especially in the financial time series where daily observations with high frequency are present. In this study, the volatility series of bitcoin price was created by using the ARCH model which was introduced by Engle (1982) considering the conditional variance.

In order to determine the ARCH (p) model for ΔP_t which is the first difference of the natural logarithm of daily bitcoin price, ARMA (p, q) model should be first estimated.

$$\Delta P_t = \delta + \sum_{i=1}^{p} \beta_i \Delta P_{t-i} + \sum_{i=1}^{q} \alpha_i \mu_{t-i} + \mu_t \qquad (1)$$

In equation (1) above, p and q are autoregressive (AR) and moving average (MA) degrees, respectively. In this equation, it is assumed that ΔP_t is covariance stationary and μ has a white noise process with variance σ_t^2. The error term μ_t obtained from the ARMA (p, q) model is subjected to the ARCH-LM test. The auxiliary regression model for the ARCH-LM test is as follows.

$$\mu_t^2 = \alpha_0 + \alpha_1 \mu_{t-1}^2 + \alpha_2 \mu_{t-2}^2 + \ldots + \alpha_p \mu_{t-p}^2 \qquad (2)$$

For the ARCH effect in equation (2) above, the null hypothesis $\alpha_1 = \alpha_2 = \ldots = \alpha_p$ must be tested. For this hypothesis, the relevant test statistic is calculated as T^*R^2. Here T represents the number of observations and R^2 refers to the explanatory power of the auxiliary regression equation. If there is an ARCH effect in the series, this effect can be eliminated by the ARCH (p) produced from ARMA (p, q) model.

$$\sigma_t^2 = \beta_0 + \sum_{i=1}^{p} \beta_i \mu_{t-i}^2 \qquad (3)$$

Constraints for ARCH (p) model in equation (3) are $\beta_0 > 0$, $\beta_i > 0$ (I = 1, 2, … p) and $\sum_{i=1}^{p} \beta_i < 1$.

However, in some cases, conditional variance is not only a function of lags of error term, but also its own lags. In this case, GARCH (p, q) model developed by Bollerslev (1986) is used to create volatility series for DLP_t.

$$\sigma_t^2 = \beta_0 + \sum_{i=1}^{p} \beta_i \mu_{t-i}^2 + \sum_{i=1}^{q} \alpha_i \sigma_{t-i}^2 \qquad (4)$$

In GARCH (p, q) model (4), in addition to the constraints of ARCH (p) model, the constraints are $\alpha_i > 0$ (I = 1, 2, … q) and $\sum_{i=1}^{p} \beta_i + \sum_{i=1}^{q} \alpha_i < 1$.

The standard GARCH model fails to capture the asymmetric effect in the variance structure. In financial transactions, investors can react differently to good news and bad news. Nelson (1991) developed the exponential GARCH (EGARCH) model to determine the asymmetric effect. Model EGARCH (1,1) is given in equation (5) below.

$$\ln(\sigma_t^2)=\beta_0+\beta_1\left(\frac{\mu_{t-1}}{\sigma_{t-1}}\right)+\gamma_1\left|\frac{\mu_{t-1}}{\sigma_{t-1}}\right|+\alpha_1\ln(\sigma_{t-1}^2) \qquad (5)$$

Since the dependent variable σ_t^2 is the natural logarithm in the EGARCH model, the coefficients in the model can be negative. The EGARCH model also captures leverage effect. If $\frac{\mu_{t-1}}{\sigma_{t-1}}$ is positive, the effect of shocks on conditional variance is equal to $\beta_1+\gamma_1$. Otherwise it will be equal to $-\beta_1+\gamma_1$.

After getting volatility series, the rolling window causality test developed by Hill (2007) was used to examine the causal relationship between bitcoin price volatility and trading volume. The Rolling window causality test is based on traditional causality tests. Toda-Yamamoto (1995) causality test was employed to determine the possible causal relationships between daily price volatility (PV) and daily trading volume (V) of bitcoin. Toda-Yamamoto causality test is as shown in equations (6) and (7).

$$V_t=\lambda_1+\sum_{i=1}^{k}\beta_{1i}V_{t-i}+\sum_{i=k+1}^{k+d_{max}}\beta_{2i}V_{t-i}+\sum_{i=1}^{k}\alpha_{1i}PV_{t-i}+\sum_{i=k+1}^{k+d_{max}}\alpha_{2i}PV_{t-i}+\mu_{1t} \qquad (6)$$

$$PV_t=\lambda_2+\sum_{i=1}^{k}\delta_{1i}PV_{t-i}+\sum_{i=k+1}^{k+d_{max}}\delta_{2i}PV_{t-i}+\sum_{i=1}^{k}\theta_{1i}V_{t-i}+\sum_{i=k+1}^{k+d_{max}}\theta_{2i}V_{t-i}+\mu_{2t} \qquad (7)$$

In Equations (6) and (7), k represents the lag length for dependent and independent variables. dmax is the maximum integrated degree of the variables. B_I, α_I, δ_I, θ_I are coefficients of the variables. Λ_1 and λ_2 represent constant terms.

In equation (6), the null hypothesis that V is not the cause of PV is as follows.

$$H_0:\ \alpha_{1i}=0 \qquad (8)$$

Similarly, in equation (7), the null hypothesis that PV is not the cause of V is as follows.

$$H_0 : \delta_{1i} = 0 \tag{9}$$

Wald test statistics are performed to determine whether the null hypothesis in (8) and (9) are rejected or not. As known, the entire sample set is not used in the rolling window causality test. On the contrary, a sample size smaller than the sample size (window width) is performed to determine the causality analysis. In the first window, a causality analysis is carried out from the first observation until the last observation of the window width. Then the next window is moved, in which the first observation is deleted and the observation after the last observation of the window width is added and the causality analysis is repeated. This process continues until the last observation in the window width is the last observation of the entire sample set.

3. Findings

In the study, ARCH/GARCH approach was employed for producing the price volatility series. Prior to ARC/GARCH estimation, the stationarity characteristics of the relevant series were examined by using Augmented Dickey and Fuller (ADF) unit root test. The related test statistics are presented in Table 1. As a result of the ADF unit root test, it was found that both trading volume and price volatility are stationary in their first differences.

Table 1

Unit root test results

	ADF-t Statistics	
Variable	**Constant**	**Constant and Trend**
P	-0.387	-2.281
V	-0.423	-2.991
ΔP	-12.89^{***}	-12.93^{***}
ΔV	-11.826^{***}	-11.847^{***}

*Note: ***, ** and * indicate that the related statistics is statistically significant at 1%, 5% and 10%, respectively. Δ implies that the related variable is first differenced. P is logarithm of bitcoin price and V is logarithm of trading volume.*

After the ADF unit root test, the ARMA structure of the logarithmic difference of the bitcoin price was determined by information criteria. Based on Akaike information criterion (AIC), ARMA (4,4) model was found to be the most appropriate for 49 ARMA models. According to the ARCH-LM test in Table 2, which presents the results of the ARMA (4,4) model, there is ARCH effect in the price series at 1% significance level. Due to the ARCH effect in the series, in this study the volatility series was created with ARCH / GARCH models.

Table 2

ARMA (4.4) Model for Bitcoin price

Dependent Variable: **ΔP**	
Constant	0.001 (0.948)
AR(1)	0.033 (1.259)
AR(2)	1.009*** (41.228)
AR(3)	0.077*** (3.154)
AR(4)	-0.935*** (-36.680)
MA(1)	-0.034* (-1.673)
MA(2)	-1.035*** (-54.015)
MA(3)	-0.053*** (-2.753)
MA(4)	0.959*** (48.637)
AIC	-3.644
F-Statistics	3.503***
Breusch-Godfrey Autocorrelation Statistics	0.121 [0.728]
ARCH-LM Test Statistics	112.544*** [0.000]

*Note: ***, ** and * indicate that the related statistics is statistically significant at 1%, 5% and 10%, respectively. The values in parentheses are t-statistics. Values in square brackets are the probability values.*

After determining the appropriate ARMA model, ARCH, GARCH and EGARCH models were estimated from ARMA (4,4) model, separately. According to both parameter constraints and AIC, ARCH (3), GARCH (2,1) and EGARCH (1,1) models were found to be the most suitable models. The results of these models are given in Table 3. EGARCH (1.1) model among them is the most appropriate model according to the AIC. Therefore, variance series produced from EGARCH(1,1) was used to be price volatility series in the causality test.

Table 3

ARCH Model results for Bitcoin price

	ARCH(3)	**GARCH(2, 1)**	**EGARCH(1, 1)**
Dependent variable	σ_t^2	σ_t^2	$\ln(\sigma_t^2)$
Constant	0.001*** (38.613)	0.001*** (11.678)	-0.512*** (-15.325)
μ_{t-1}^2	0.133*** (8.12)	0.133*** (10.662)	
μ_{t-2}^2	0.044*** (3.1)	0.044** (2.288)	
μ_{t-3}^2	0.044*** (5.419)		
σ_{t-1}^2		0.533*** (14.56)	
$\left\lvert\frac{\mu_{t-1}}{\sigma_{t-1}}\right\rvert$			0.256*** (17.823)
$\frac{\mu_{t-1}}{\sigma_{t-1}}$			-0.02** (-2.381)
$\ln(\sigma_{t-1}^2)$			0.949*** (238.025)
AIC	-3.746	-3.772	-3.875
ARCH-LM Test Statistics	2.699 [0.1]	0.986 [0.321]	1.258 [0.262]

*Note: ***, ** and * indicate that the related statistics is statistically significant at 1%, 5% and 10%, respectively. The values in parentheses are t-statistics. Values in square brackets are the probability values.*

The volatility series derived from the EGARCH model above was investigated with the ADF unit root test before causality test. It was found that price volatility series is stationary in its level. Previously, the trading volume of bitcoin has been found to be stationary in its first difference. Therefore, the possible causal relationships between bitcoin price volatility and trading volume was explored by using Toda-Yamamoto (1995) causality approach. The window widths in Toda-Yamamoto causality test are used to be 50, 100 and 200. The optimal lag lengths in the models were identified by AIC. Gauss codes written by Hill (2012) were used to detect the dynamic structure of the possible causal relationships the two variables. The analysis used both Wald and bootstrap statistics resolved 5000 times.

The rate of rejection of the null hypothesis, which states that there is no causal relationship between trading volume and price volatility is shown in Table 4. The null hypothesis that there is no causality running from price volatility to trading volume in the rolling window analysis is rejected in the 50, 100 and 200 window widths by 33.28%, 43.26% and 41.29% respectively. On the other hand, the null hypothesis that the causality does not run from the trading volume to the price volatility is rejected at the same window widths as 63.95%, 84.68% and 89.02%, respectively.

According to the Bootstrap test statistics, the null hypothesis that implies no causality running from volatility to volume is rejected in the 50, 100 and 200 window widths by 33.71%, 41.76% and 42.04%, respectively. The null hypothesis that there is no causal relationship from volume to volatility is also rejected at the same window widths as 47.63%, 73.39% and 87.98%, respectively. The rate of rejection of the null hypothesis, which states that there is no causal relationship from price volatility to volume, is almost the same in Wald and Bootstrap techniques. This finding is independent of the window widths. However, Wald and Bootstrap techniques differ in terms of the rejection rate of the null hypothesis that there is no causal relationship from volume to price volatility. When the window width is 50 and 100, the rate of rejection of the null hypothesis in the Bootstrap method is less than the Wald method. If the window width is 200, the rejection rate of the null hypothesis is the same in both techniques. According to the test statistics given in Table 4, there is a bidirectional causality relationship between price volatility and trading volume. However, this causal relationship is stronger from trading volume to price volatility.

Table 4

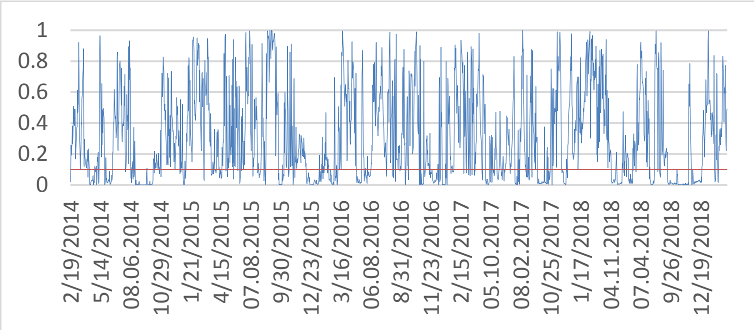
The rate of rejection of null hypothesis at 10% significance level

Window Width	No causality from price volatility to trading volume		No causality from trading volume to price volatility.	
	Wald	**Bootstrap**	**Wald**	**Bootstrap**
50	33.28%	33.71%	63.95%	47.63%
100	43.26%	41.76%	84.68%	73.39%
200	41.29%	42.04%	89.02%	87.98%

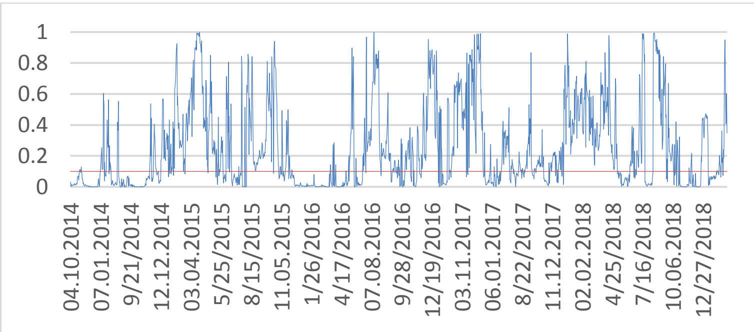
Graph 1

Price volatility does not Granger cause trading volume

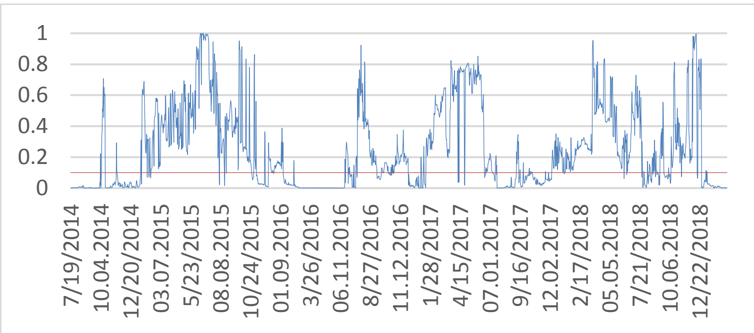
Panel a: Window width of 50



Panel b: Window width of 100



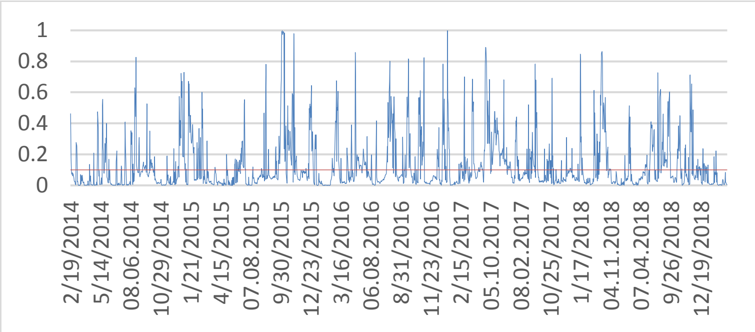
Panel c: Window width of 200



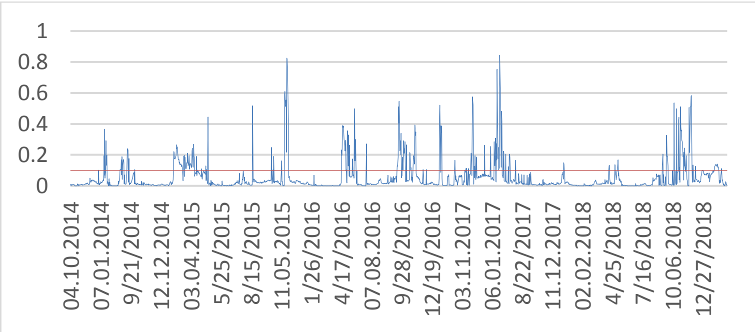
Graph 2

Trading volume does not Granger cause price volatility

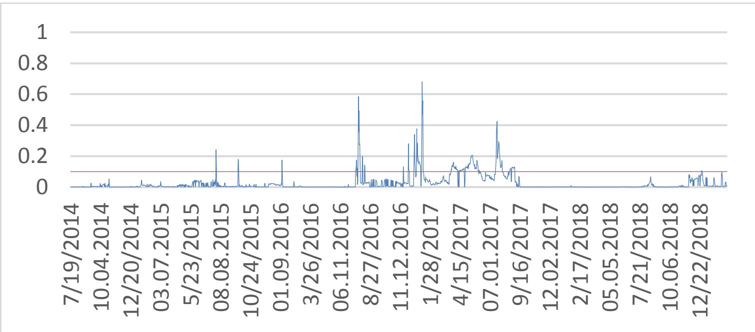
Panel a: Window width of 50



Panel b: Window width of 100



Panel c: Window width of 200



Graph 1 shows the bootstrap p values for the null hypothesis that there is no causal relationship from price volatility to trading volume for 50, 100 and 200 window widths. From the related figure, it is seen that the causal relationship from price volatility to trading volume in 50 window widths is not continuous. However, in the case of an increase in the width of the window, this causality becomes more continuous. Especially in the analysis using 200 window width, the causality from price volatility to trading volume is continuous between March 2016 and June 2016. In addition, the bootstrap p values for the null hypothesis that the trading volume does not cause price volatility are presented in Graph 2 for window widths 50, 100 and 200. According to this graph, the causal relationship from trading volume to price volatility is not continuous for the window width of 50. However, from the same graph, it can be observed that the causality from volume to volatility is strengthened and becomes more permanent if more window width is used. This continuous causal relationship appears to take place almost over the entire period of the window width of 200. When both graphs are evaluated together, it can be concluded that there is a bidirectional causal relationship between price volatility and trading volume for bitcoin. However, from all panels of both graphs it can be detected that the causal relationship especially from trading volume to the price volatility is stronger and more continuous.

4. Conclusion

The relationship between price volatility and trading volume in any asset market has been a subject of debate in the finance literature for many years. There are two basic hypotheses between the two related variables. The mixture of distribution hypothesis does not predict any causal relationship between the two variables, whereas the sequential information arrival hypothesis states that there is a bidirectional causal relationship between the two variables. The related hypotheses were generally tested on the stock markets in the empirical literature and the findings mostly supported the validity of the sequential information arrival hypothesis.

In the present study, in order to determine whether the findings on stock markets are valid for cryptocurrency market, the possible causal relationships between the price volatility and the trading volume of bitcoin were investigated by using the rolling window causality method. Bitcoin price volatility is produced under EGARCH (1,1) model. According to the findings obtained under three different window widths, there exists a bidirectional causal relationship between two variables. The causal relationship from volume to price volatility is stronger than the causal relationship from volatility to volume. It means that a new information coming to the bitcoin market is not available at the same moment to all buyers and sellers and hence sometimes price volatility causes trading volume and sometimes volume causes price volatility. In addition, the contemporaneous correlation coefficient between the two variables is positive and statistically significant. Both the causality and correlation analysis results show that the sequential information arrival hypothesis in the bitcoin market is valid. Ultimately, the bitcoin market is not a market within the scope of efficient markets hypothesis.

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