ROLE OF EARLY WARNING SYSTEMS IN PREDICTING THE STOCK PRICE CRISIS: WHAT WE LEARNT FROM GRASSHOPPER AND ANTS FABLE

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Abstract

Early warning systems are too important tools in predicting the crisis in financial institutions say banks and stock markets. A consequence of crashes in a specified stock or stock market is financial crisis. This paper considers designing an early warning system based on random walk theory and maximal inequality. First, mathematical tools are presented, and the early warning system is designed, then some real data sets are analysed. The performance of system is evaluated by some different criteria. After it, using a dynamic programming approach, a modified version of mentioned early warning system is proposed. Finally, a conclusion section is given.

Keywords: Crash indicator; Dynamic programming; Early warning system; Stock market; Variance ratio

JEL Classification: G21

1. Introduction

The famous grasshopper and ants fable teaches us to worry (now in one warm spring day) about chance of occurring bad winter and food shortage (happening bad events) in future. There, grasshopper learnt it is necessary to prepare for tomorrow and work today for what you need in future. Like this tale, a similar case exists for financial environments and institutions. Although, there is no crises and crash in a specific bank, insurance company or financial institution, however, there is no guarantee for a bad event in even near future, similar event could happen for grasshopper. There are many types of

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financial crisis including banking crisis, currency crisis, speculative bubbles crashes, wider economic crisis, and international financial crisis. Most important causes of financial crisis are regulatory failures, uncertainty and herd behaviour, asset-liability mismatch, the risk of bankruptcy because of leverage, contagion and recessionary effects (see, Friedman and Posner,2010). One of dramatic type of financial crisis is the stock market crash at which stock prices drop rapidly and suddenly. It leads to speculative bubbles and economic crisis (see, Sornette, 2017).

Different economic crises such as banking, financial, and currency lead to high economic costs and have negative impact to whole society. Development of early warning systems could help in prevention of economic and business crisis, while they present a systematic forecast of unwanted events. Early warning systems are used primary for detecting crises before damage has been made and for reducing false alarms of possible crisis.

There are different approaches to develop a EWS; such as signal approach, binary classification tree and logit model, see Cashin and Duttagupta (2008). In its simplest version, the early warning systems (EWS) are warning systems designed to detect imminent disasters in financial systems. There are interesting EWS methods for stock market crashes say signal extraction methods, logit models, Bayes network, hidden Markov models, switching linear dynamic systems, naive Bayes switching linear dynamic system (see, Dabrowski *et al*., 2016). Acar *et al*. (2011) detected the stock market crashes using adaptive neuro fuzzy inference system (ANFIS) model. EWS's are also designed using data mining and machine learning techniques like artificial neural network, support vector machine and decision trees like classification and regression trees (CART) and chi-squared automatic interaction detection (CHAID). For comprehensive review in EWS for financial crisis, see Acar *et al*. (2011). For an overview of empirical studies on EWS, see Ivashina and Scharfstein (2010) and Monnasoo & Mayes (2009). They have used different macroeconomic variables. Indeed, the economic variables associate crises with the conditions of financial sectors, external sectors, and real sectors. With regard to the latter, a few studies, see Walter and Willett (2012).

Acar (2010) studied the behaviour of stock market crash index $C_t = x_t s_t^2$ in Istanbul stock exchange (ISE) where x_t is the ISE national

100 index, $p_t = \frac{x_t - x_{t-1}}{x_{t-1}}$, s_t^2 is ten-day rolling variance of p_t and $r_t = \frac{s_t^2}{s_{t-1}^2}$ is the variance ratio index. Each financial time series has two important components which increase at each component results financial crises.

These components are the level of time series which is x_t and its variance s_t^2 (or volatility, if, s_t is used in C_t, instead of s_t^2). Thus the combined multiplicative index of these components is $C_t = x_t s_t^2$.

Notice that $C_t = C_{t-1} r_t (1 + p_t)$. Let $c_t = \log{(C_t)}$. Then,

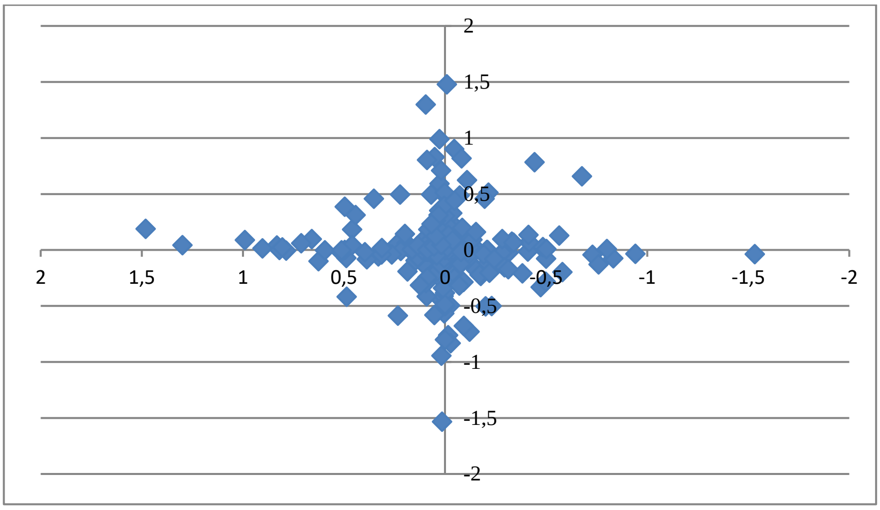
$$c_t = c_{t-1} + \log(r_t) + \log(1 + p_t) \cong c_{t-1} + \log(r_t) + p_t = c_{t-1} + u_t.$$

The last equation uses identity $\log{(1 + x)} \cong x$ for small x. This equation defines a random walk structure for c_t, the logarithm (in natural base) of crash index. Here, the behaviour of increment process $u_t = \log(r_t) + p_t$ is studied. First, the $\log(r_t)$ is considered. The real data set is the daily stock price of Apple Corporation for one year time period of 2 March of 2017 to 2 March of 2018. Although, it is better to do this result for the stock market index, however, a single share such as *Apple co*. may have more fluctuations which shows the validity of theoretical result better than the stock market index. In that case, because of diversity of different stocks, their volatilities may offset by each other and researcher should be wait to see a considerable crises.

The following figure shows the scatter plot of $\log(r_{t-1})$ versus $\log(r_t)$. It is seen that there is no serial correlation in $\log(r_t)$.

Figure 1

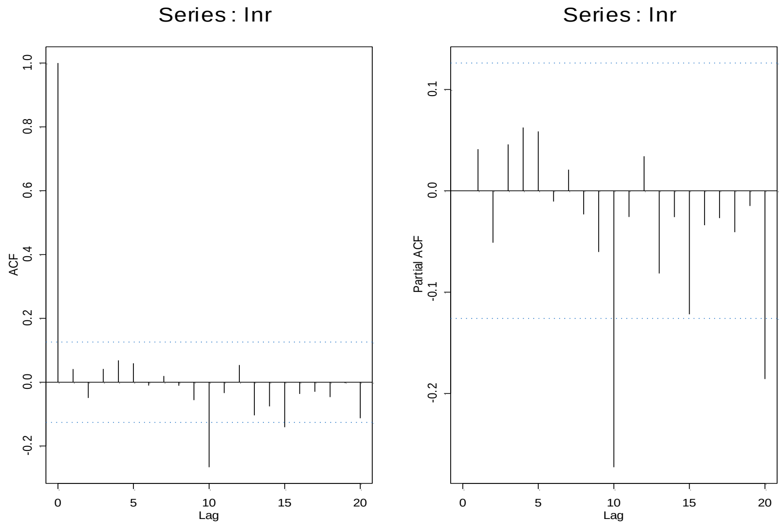
Plot of $\log(r_t)$ vs. $\log(r_{t-1})$



The autocorrelation (ACF) and partial autocorrelation (PACF) functions are plotted as follows:

Figure 2

ACF and PACF of $\log(r_t)$

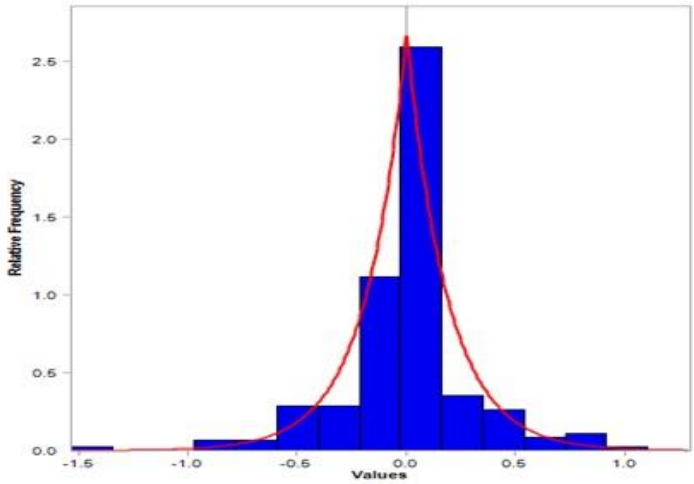


Again, it is seen that there is no time series pattern of $\log(r_t)$. As follows, the cumulative sum and cumulative sum of square plots of $\log(r_t)$ are given to check the existence of change points in mean or variance of $\log(r_t)$ which shows that there is no change point in mean and variance of $\log(r_t)$.

This facts show that the $\log(r_t)$ is an independent sequence of random variables. To find the distribution of $\log(r_t)$ it is seen that a Laplace distribution is best fitted to data with location parameter almost equal to zero and scale parameter 0.2637. It is done using the *Modelrisk* software.

Figure 3

Histogram and fitted density of $\log(r_t)$



The second part of u_t is p_t at which there are many famous models say geometric Brownian motion (GBM) which is fitted well to x_t. Next, the existence of correlation between p_t and $\log(r_t)$ is surveyed. The sample correlation is 0.025 which implies the independence of p_t and $\log(r_t)$. Thus, $c_t = \log(C_t) = c_{t-1} + u_t$ and $u_t = \log(r_t) + p_t$ at which $\log(r_t)$ has central Laplace distribution with scale parameter 0.2637 independent of p_t where x_t is a GBM process with parameters $\mu = 0$ and $\sigma = 0.0123$. Throughout the paper, this structure is kept fixed and using standard theorems in random walk theory, some useful features of stock market crash index C_t are derived.

This paper is organized as follows. In the next section, some mathematical aspects of this model are derived. Some real data sets are studied in section 3 and the performances of EWS's are surveyed in section 4. Dynamic programming solution is proposed in section 5. Conclusions are given in section 6.

2. Mathematical results

Let x_t denote the price of a stock (stock market index or a specified stock) at time t, and its return be p_t, L-day rolling variance (volatility) of p_t be s_t^2, and $r_t = \frac{s_t^2}{s_{t-1}^2}$ be the variance ratio index. Then, the logarithm (in natural base) of stock crash index is given by $c_t = \log(C_t) = c_{t-1} + u_t$ at which $u_t = \log(r_t) + p_t$ where p_t's are independent with common $N(0, \sigma^2)$ distribution and $z_t = \log(r_t)$ has Laplace distribution with density $\frac{1}{2\theta} e^{-\frac{|z|}{\theta}}$ $-\infty < z < \infty$. This method can be considered as a signaling approach to EWS 's approaches.

It is easy to see that $E(u_t) = 0$ and $cov(u_t, u_{t+h}) = 0$ for $h \neq 0$ and $var(u_t) = \sigma^2 + 2\theta^2 < \infty$. Let γ denote the threshold at which $C_t > \gamma$ is equivalent to stock (market) crash. Then, for large t's, the probability of crash, using the central limit theorem (CLT), is given by

$P(C_t > \gamma) = p(c_t > \log(\gamma)) \approx 1 - N\left(\frac{\log(\gamma)}{\sqrt{t(\sigma^2+\theta^2)}}\right)$, where N is the standard normal distribution function. When, p_t's are independent with time varying $N(0, \sigma_t^2)$ distributions, then

$$p(c_t > \log(\gamma)) \approx 1 - N\left(\frac{\log(\gamma)}{\sqrt{\sum_{i=1}^{t}(\sigma_i^2 + 2\theta^2)}}\right).$$

Also, since $E(u_t) = 0$, thus, c_t is a martingale. Let the stopping time τ_γ be first time at which C_t passes γ, indeed,

$$\tau_\gamma = \inf\{t; C_t > \gamma\} = \inf\{t; c_t > \log(\gamma)\}.$$

Thus,

$$P(\tau_\gamma \leq t) = P(max_{0\leq s\leq t} c_t \geq \log(\gamma)) \leq \frac{E(c_t^2)}{(\log(\gamma))^2} = \frac{t(\sigma^2 + 2\theta^2)}{(\log(\gamma))^2}.$$

For the last inequality, the Doob martingale inequality, see Bjork (2009), is used. When, p_t's are independent with time varying $N(0, \sigma_t^2)$ distributions, thus, using the maximal inequality for partial sum, it is seen that

$$P(\tau_\gamma \leq t) \leq \frac{\sum_{i=1}^{t}(\sigma_i^2 + 2\theta^2)}{(\log(\gamma))^2}.$$

Suppose that the upper bound of $P(\tau_\gamma \leq t)$ is considered as the probability of crash, therefore, $P(\tau_\gamma = t) \propto \frac{\sigma_t^2 + 2\theta^2}{(\log(\gamma))^2}$ for discrete time points t's. For example, assume that $t = 1, \dots, n$, therefore, $P(\tau_\gamma = t) = \frac{\sigma_t^2 + 2\theta^2}{\sum_{i=1}^{n}(\sigma_i^2 + 2\theta^2)}$.

Remark 1. Notice that, using Doob inequality, $P(\tau_\gamma \leq t) \leq \frac{E(f(c_t))}{\mathrm{f}(\log(\gamma))}$, for every strictly increasing positive function f. This stylized fact changes the values of probabilities, however, here, the probability is a tool that its large values at a specified time point shows the possible potentially crash and it is enough and it isn't important which type of probability function (what is f) is applied.

Remark 2. Notice that $P(\tau_\gamma \leq t) = P(max_{1\leq j\leq t}|c_j| > \log(\gamma)) \leq \frac{1}{(\log(\gamma))^2} E(c_t^2)$. Then, to bound the probability of crash, let

$$\frac{1}{(\log(\gamma))^2} E(c_t^2) \leq \varepsilon,$$

for some predefined threshold ε. Then, $E(c_t^2) \leq \varepsilon(\log(\gamma))^2$. An estimate for $E(c_t^2)$ is the average of $c_k^2, k = t-2, t-1, t$, that is, $\overline{c^2}_t$. Let

$$\tau_\gamma^* = \inf\{t; \overline{c^2}_t > \varepsilon(\log(\gamma))^2\}.$$

This new stopping time also defines a new index for timing the crash of stock (market) index. To find γ or ($\log(\gamma)$), notice that $E(c_t^2) = \sum_{j=1}^{t} E(u_j^2) = \sum_{j=1}^{t}(2\theta^2 + \sigma_i^2)$. Thus, $\frac{E(c_t^2)}{(\log(\gamma))^2} < \varepsilon$, implies that $\log(\gamma) > \sqrt{\frac{\sum_{j=1}^{t}(2\theta^2+\sigma_i^2)}{\varepsilon}}$. When, $\sigma_i^2 = \sigma^2$, then $\log(\gamma) > \sqrt{\frac{t(2\theta^2+\sigma^2)}{\varepsilon}}$. When, study period is $t = 1, \ldots, T$, then

$$\log(\gamma) = \sqrt{\frac{T(2\theta^2 + \sigma^2)}{\varepsilon}}$$

Another estimate for $E(c_t^2)$ is to substitute it by the sample variance of c_t.

3 Real data sets

Here, the above methods like probabilities (CLT, stopping time based) and criterion of remark 2 are studied for some real data sets.

(*a*) *Apple Co. data set*

To plot CLT probabilities, it is assumed $\log(\gamma) = 1$, hypothetically.

Figure 4

CLT based probability of crash

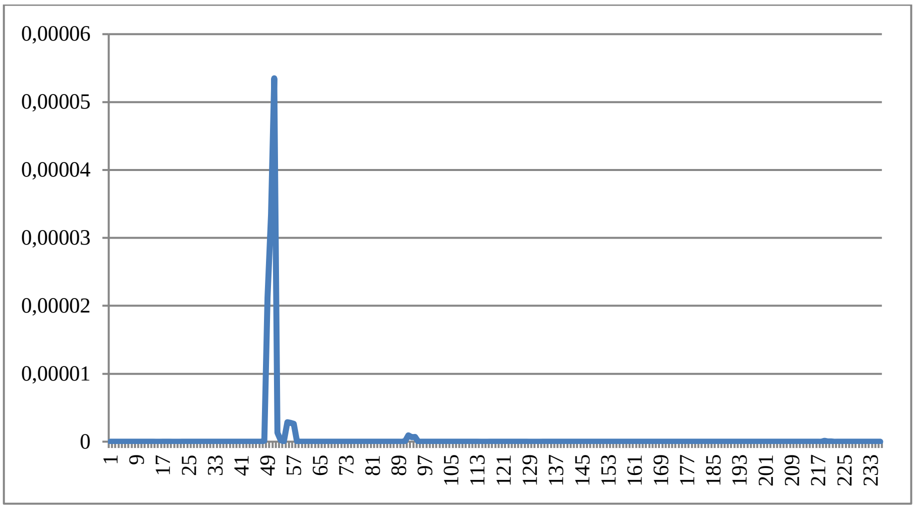
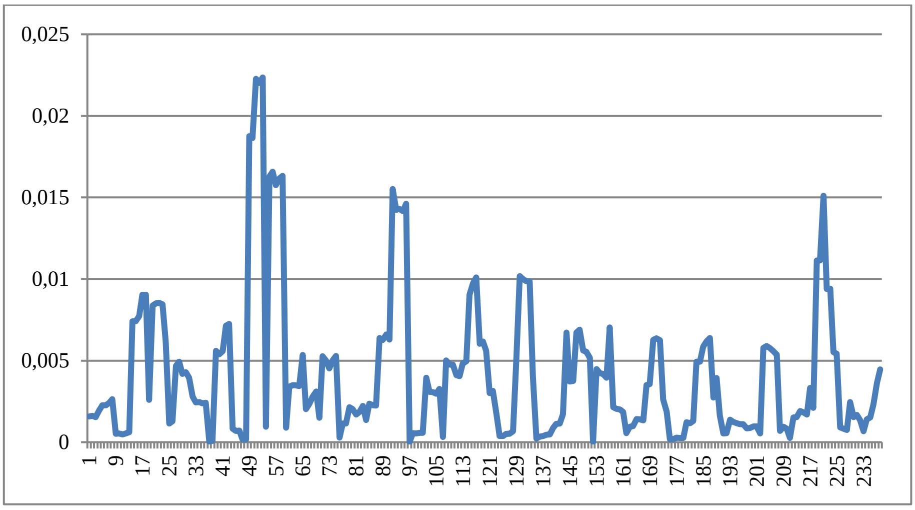


Figure 5

Stopping time-based probability of crash



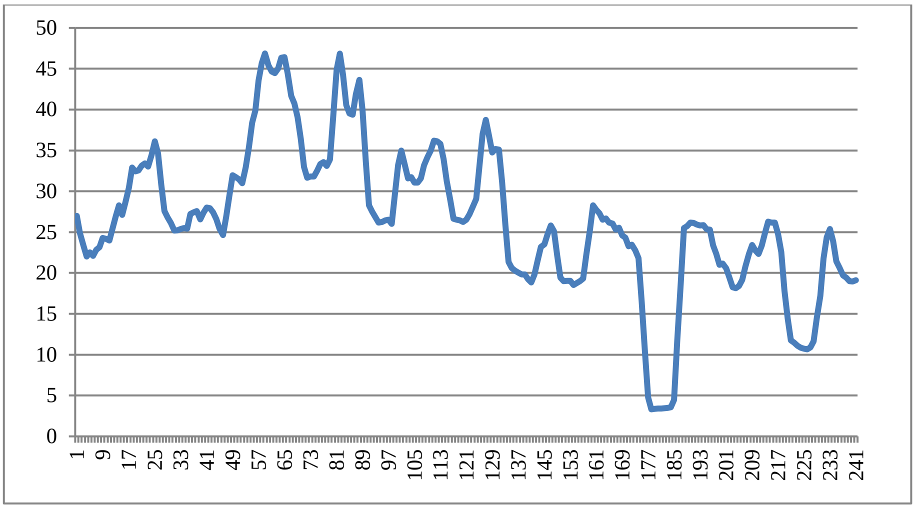
Each probability indicates that there is a tendency to crash in time point numbered 49 which corresponds to the 26 May 2017.

(b) *General Motors (GM) company*

Using the method of remark 2, the daily stock price of General Motors (GM) company is studied during 9 March 2017 to 9 March 2018. The following plot shows the time series plot of $\overline{c^2}_t$, where the ten days variance are computed and then $\overline{c^2}_t$ are calculated.

Figure 6

Time series plot of $\overline{c^2}_t$



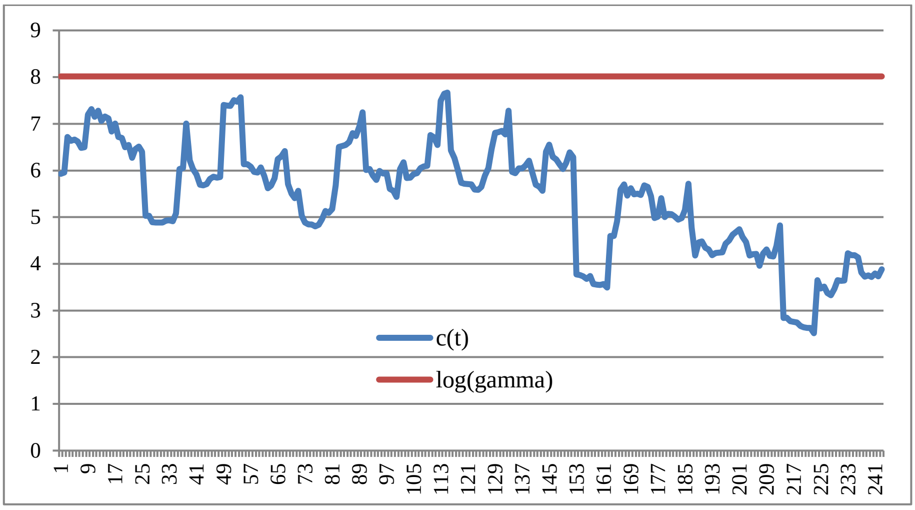
It is seen that the $\overline{c^2}_t$ goes up around $t = 56$ and 78 which are 15/6/2017 and 18/7/2017, respectively.

(c) *Intel Company*

Here, the Intel stock price is studied during 9 March 2017 to 9 March 2018. The sample variance of c_t is 0.6424 and assuming $\varepsilon = 0.01$, then $\log(\gamma) = 8.015$. The following figure gives the time series plot of c_t in the presence of threshold 8.015. It is seen that there is no chance for crashing of stock price of Intel Co.

Figure 7

Time series plot of c_t



(*d*) *JPMorgan Chase & Co*

Here, the empirical distribution of $\tau_\gamma = \inf\{t > 0; c_t > \log(\gamma)\}$ is obtained as follows. To this end, the *JPMorgan Chase & Co* stock price data set is studied during 9 March 2017 to 9 March 2018.

The $var(c_t) = 0.4433$ for sub-period of 9 March 2017 to 28 July 2017. Assuming $\varepsilon = 0.02$, then $\log(\gamma) = 4.71$.

A geometric Brownian motion is fitted to the stock price of *JPMorgan Chase & Co* with $\mu = 0.001008$ and $\sigma = 0.011$. Running 1000 iterations of Monte Carlo simulation yields the following histogram of $\tau_\gamma/257$. The mean and standard deviation are 0.1525 and 0.1536, respectively and skewness and kurtosis measures are 1.84 and 7.13. Fitting a generalized extreme value (GEV) distribution gives the estimated location, scale, and shape parameters 0.1018, 0.1038 and 0.4098, respectively.

Figure 8

Histogram of $\tau_\gamma/257$

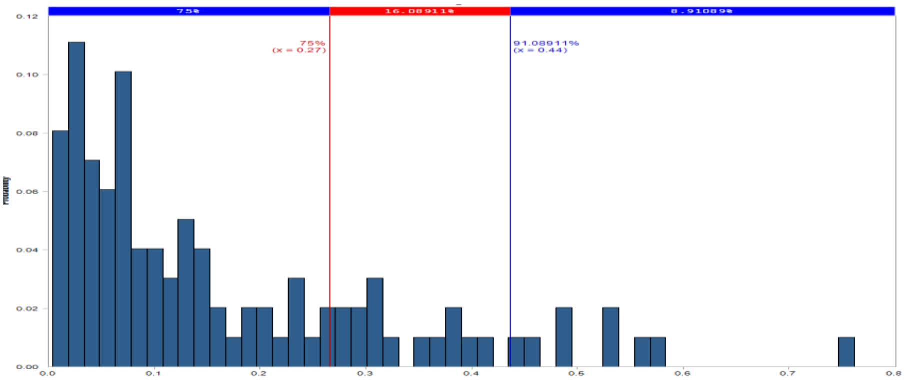
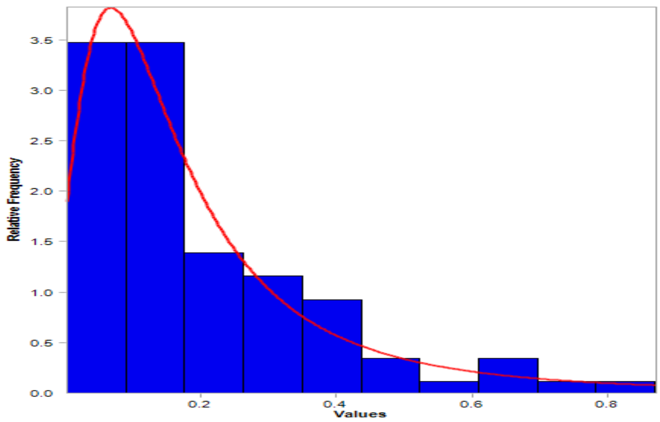


Figure 9

Density estimate of $\tau_\gamma/257$



4. Performances

Here, three measures are proposed to study the performance of above-mentioned procedures. Performance criteria are the probabilities of type I and type II errors measures and signal to error criterion.

4.1 Type I error. Again, consider the historical stock price of Intel Company, part c, section 3. Let $\varepsilon = 0.01$, then $\log(\gamma) = 8.015$. Suppose that T is 9 March 2018 and it was seen that there is no crisis during this period of study. Notice that the probability of type I error, that is $\alpha = P(\tau_\gamma \leq T) = P(max_{1\leq t\leq T}|c_t| > \log(\gamma))$. As follows, the second probability is simulated using the Monte Carlo simulation. To this end, the path of process of stock price of Intel Company x_t is simulated using the GBM process with $\mu = 0.00149$ and $\sigma = 0.01478$ and then ten-day variances are computed and thus the c_T is simulated. In this case, $\alpha = 0$. The following table gives several values of α's based on several choices for ε.

Table 1

Type I errors for various selects of ε

ε	0.005	0.01	0.015	0.02
$\log(\gamma)$	11.34	8.015	6.544	5.66
α	0	0	0.27	0.997

It is seen that for a small change in ε, unfortunately, α grows too fast. However, if one selects too small values for ε, no crisis is detected, spuriously. This shows the inadequacy of upper bound $\log(\gamma) = \sqrt{\frac{\widehat{var}(c_t)}{\varepsilon}}$ where $\widehat{var}(c_t)$ is the empirical estimate of $var(c_t)$. To overcome this difficulty, it is enough to find the null hypothesis (of no stock crisis) distribution of $max_{1\leq t\leq T}|c_t|$ and find $\log(\gamma)$ such that $P(max_{1\leq t\leq T}|c_t| > \log(\gamma)) = \alpha$.

The following table gives the values of $\log(\gamma)$ based on various selects for α.

Table 2

Various values of $\log(\gamma)$

α	0.01	0.025	0.05	0.1
$\log(\gamma)$	7.387	7.173	7.032	6.852

4.2 Type II error. Notice that the probability of type II error is $\beta = P(max_{1\leq t\leq T}|c_t| < \log(\gamma))$ when there is at least one crisis in stock price, in reality. Again, the following table gives the β. To induce stock crisis, during simulating stock price of Intel Company x_t, its values are changed to $x_t - 20$ for $t = 100, \ldots, 106$ and to $x_t - 26$ for $t = 216, \ldots, 223$. It is seen that values of β's are too close to α's, surprisingly.

Table 3

Various values of β

α	0.01	0.025	0.05	0.1
β	0.015	0.023	0.047	0.098

4.3 Noise to signal ratio. Following Bryde-Erichsen (2016), the noise to signal ratio (NSR) is defined as the proportion of false positive rate to true positive rate, that is

$$NSR = \frac{\frac{FP}{FP+TN}}{\frac{TP}{TP+FN}},$$

where FP, FN, TN, TP are elements of confusion matrix as follows

Table 4

Confusion matrix and its elements

Decision/Real situation	Crisis exists	No crisis
Crisis exists	TP	FP
No crisis	FN	TN

Here, the data set of General Motors (GM) company, part (b) of section 3, is used to compute the NSR criterion. First, a GBM process is fitted to stock price of this company with $\mu = 000107$ and $\sigma = 0.0177$. Then, $max_{1 \le t \le T} |c_t|$ is simulated using Monte Carlo simulation to compute the 99 percent quantile of this random variable which is 7.25. Here, supposing there is no crisis, then, $FN = 100$ and $TN = 100$. For constructing a crisis, a decline in μ is made and it is assumed that after $t = 150$ (after 10 October 2017) μ is changed to 0.00005. Again, it is seen that $TP = 749$ and $FP = 251$.

The confusion matrix is given as follows

Table 5

Confusion matrix for GM company

Decision/Real situation	Crisis exists	No crisis
Crisis exists	749	251
No crisis	100	900

Here, the NSR is 0.247 which is a small number and shows the accuracy of method.

5. Dynamic programming solution

In this section, a dynamic programming approach is proposed for designing the early warning system. Notice that $c_t = c_{t-1} + u_t$, $u_t = \log(r_t) + p_t = p_t + z_t = g(p_t, z_t)$ where $z_t = \log(r_t) = \log\left(\frac{s_t^2}{s_{t-1}^2}\right)$. Here, it is assumed that crisis at stock price x_t occurs at time t at which z_t is large. Thus, a modified c_T criterion is given by

$$c_T = max_{\{z_j\}_{j=1}^{T}} \sum_{j=1}^{T} g(p_j, z_j).$$

The Bellman equation is given by

$$c_t = max_{\{z_{t+1}\}}\{c_{t+1} + g(p_{t+1}, z_{t+1})\}.$$

Thus, a backward induction is used to obtain all values of z_t and the time of crisis is estimated by τ at which $c_\tau = \max(c_t)$. Equivalently, $\mathrm{z}_\tau = \mathrm{argmax}_{1\le t\le T} \mathrm{c}(\mathrm{z}_t)$. Here, the data set of Exxon Mobil Corporation (XOM) during 16 March 2017 to 16 March 2018. Here, again, a Laplace distribution is fitted to the z_t with location and scale parameters -0.0171, 0.260425, respectively. The 99 percent quantile of simulated $\max_{1\le t\le T} c_t$ is 2.074. For simulating $\max_{1\le t\le T} c_t$, first, 7 samples are drawn of Laplace distribution with location and scale parameters -0.0171, 0.260425 and its maximum is taken. Then, this value is added to p_t and u_t is simulated. Finally, c_t is computed.

6. Conclusions

As stated in the introduction of paper, there are many approaches for seeing warning alarm soon to prevent or reduce huge losses of financial crises in different markets and institutions, such as banking, stock market, or even insurance. These approaches are mainly data mining and data science-based methods. However, the nature of stock prices and returns are probabilistic and statistical distributions and stochastic processes govern on them. Also, there are many strong techniques such as Doob-inequality in modern probability literatures. Therefore, it is interesting to use these useful results to build a EWS. To this end, in this paper, the logarithm of crash indicator (and consequently its variance) is decomposed to its lag (variance of lag) and L-rolling volatility ratio (variance of L-rolling volatility ratio). This decomposition constructs a basis for upper bounding the probability of existence of crisis defined by some suitable stopping times. Simulation results show the good performance of EWS in terms of type I and II errors.

References

1. Acar, M. (2010). Designing an early warning system for stock market crashes based on machine learning forecasting. Master Thesis. School of Natural and Applied Sciences. Bahcesehir University. Istanbul. Turkey.
2. Acar, M., Karahoca, D., and Karahoca, A. (2010). Designing an early warning system for stock market crashes by using ANFIS. Downloadable from www.igi-global.com/chapter/designing-early-warning-systemstock/73544?camid=4v1
3. Bjork, T. (2009). Arbitrage theory in continuous time. Oxford University Press.
4. Bryde-Erichsen, M. D. (2016). The signaling approach to early warning: Application for systemic banking crisis. Unpublished Master Thesis in Economics. Department of Economics. University of Oslo.
5. Cashin, P. A. & Duttagupta, R. (2008). The anatomy of banking crises. International Monetary Fund [Online], Working Papers No. 08/93. http://www.imf.org/external/pubs/ft/ wp/2008/wp0893.pdf.
6. Dabrowski, J. J., Beyers, C. and de Villiers, J. P. (2016). Systemic banking crisis early warning systems using dynamic Bayesian networks. Expert Systems with Applications 62, pp. 225-242.
7. Friedman, J. and Posner, R. A. (2010). What caused the financial crisis. University of Pennsylvania Press. Philadelphia, Pennsylvania. USA.
8. Ivashina, V. and Scharfstein, D. (2010). Bank lending during the financial crisis of 2008. Journal of Financial Economics 97, pp. 319-338.
9. Monnasoo, K. and Mayes, D. G. (2009). Explaining bank distress in Eastern European transition economies. Journal of Banking & Finance 33, pp. 244-253.
10. Sornette, D. (2017). Why stock markets crash: critical events in complex financial systems. Princeton University Press. New Jersey.
11. Walter, S. and Willett, T. D. (2012). Delaying the inevitable: A political economy approach to currency defenses and depreciation. Review of International Political Economy 19, pp. 114-139.