INVESTIGATING THE OPTIMAL EXIT TIMING AND LEVERAGE DURING THE COVID-19 CRISIS

Mohamed Ben ABDELHAMID, PhD*

Makram BELLALAH, PhD**

Abstract

This paper investigates the effectiveness of the corporate credit policies as a means of preventing market exit in the aftermath of the COVID-19 pandemic. A real options framework incorporating dynamic programming is employed to investigate the relationship between exit decisions, leverage ratio and productivity uncertainty. Our paper presents a novel approach to the exit problem in comparison to other attempts in early 2020. Taking into account the dynamics of firms, we allow for a variety of factors, such as productivity uncertainty, debt readjustment, liquidity constraints, and leverage level, to explain the optimal time for a firm to exit during the COVID-19 pandemic. Our results indicate that the corporate credit programs have a significant positive impact and suggests that a greater leverage ratio increases the likelihood of survival and delays the decision to exit.

Keywords: uncertainty, liquidity productivity, debt, real options

JEL Classification: G01; G33

1. Introduction

The coronavirus outbreak has posed the most severe challenge to economies worldwide, resulting in a historical recession with one of the most dramatic falls in modern times. The financial health of companies has been significantly impacted due to the corona-crash, with dire liquidity shortages and funding supply being of particular concern as the severity of the COVID-19 pandemic directly impacted consumer behavior and market demand. The International Monetary

* *Associated researcher, LEFMI, University of Picardie Jules Verne, Amiens, France.*

** *Professor, LEFMI, University of Picardie Jules Verne, Amiens, France.*

Fund (IMF, 2020) has identified that numerous countries have implemented various forms of financial assistance for small and medium enterprises (SMEs) to combat the effects of the COVID-19 pandemic, generally in the form of loans and loan guarantees. Equity markets have been profoundly affected, and so "debt and more debt" has become a central component of many organizations' support schemes to address the consequences of the coronavirus on companies, particularly SMEs. Nevertheless, questions remain as to how businesses should be salvaged and what type of financing would be most appropriate. Becker et al. (2020) suggested that the use of credit support is a viable means of responding to the COVID-19 pandemic, due to the importance of preserving economic sovereignty and fiscal resources, as well as the difficulty of distinguishing the damaged from the undamaged firms in the crisis due to the heterogeneity of the effects of the pandemic across industries. Subsequently, Sinagl (2020) found that differences in the effects of the pandemic on firms' revenue may be attributed to differences in consumer savings propensity and willingness to spend.

In order to regain their financial health, companies needed to secure additional liquidity to protect their value and avoid any financial difficulties. Working through liquidity issues can have an effect on the company's capital structure and financial leverage, which could potentially transform a liquidity crisis into a solvency concern. Despite the fact that numerous economists and international organizations (OCDE 2020, Moody’s 2020) claim that corporate balance sheets were already highly leveraged prior to the COVID-19 crisis, credit remains the only way to ensure their survival given the absence of internal and external funding. Boot et al. (2020) found that governmental assistance programs that rely on debt financing can increase leverage, and therefore the "default risk," but are still preferable to “no-support”. Bartik et al. (2020) further highlighted that firms with limited cash flows may have to choose between taking on additional debt or declaring bankruptcy. Megginson and Fotak (2020) described the COVID emergency as a “liquidity crisis” and they believed that the most appropriate response is to provide government financial support in the form of short-term bridge financing to sustain businesses and preserve employment. This should only be required for a few months rather than years. However, the authors conclude that rescuing distressed companies by injecting equity is more suitable than granting emergency debt. As many firms were suffering from a liquidity crisis, it was difficult to bear the additional fixed cost (interest and principal payments on debt) and additional distress risk that higher leverage would bring.

We propose a dynamic tool whose contribution, presented by this paper, is a starting point for a pragmatic methodology based on real options that can guide other researchers in studying the effectiveness of governmental credit support policy under demand uncertainty. We propose to extend Olley and Pakes' (1996) model by including a debt parameter to explore the impact of demand shock, market efficiency, and capital adjustment on the exit decision. The primary concept of Olley and Pakes (1996) is that productivity is a function of capital stock and investment. This is used to define a firm's behavior in terms of whether to exit the market or to invest through financing actions, based on a productivity threshold level. While the application of a structural approach for decision-making in real-world problems is often limited due to the need for detailed data and uncertain future scenarios, the real option approach has traditionally provided an effective model framework to analyze regular investment and exit decisions (Dixit, 1989; Dixit & Pindyck, 1994). We assume a list of assumptions in order to create an analytical solution for exit decision. By finding the optimal stopping time, expressed as a function of leverage ratio, this model captures the interaction between exit threshold and leverage level under persistent profitability uncertainty. Murto and Terviö (2014) have argued that persistent profitability implies that a firm should exit if the current revenue falls under a threshold boundary. In our standard real options model, we factor in debt adjustment costs according to Q-theory.

The paper is structured as follows: Section 2 provides a review of relevant literature. Section 3 introduces a conceptual framework for exit decisions, extending Olley-Pakes' approach to focus on financing instead of investment decisions. Section 4 outlines a simple analytical solution exit problem based on real options. In Section 5, simulation is used to analyze the numerical results of our model. Finally, Section 6 concludes the paper.

2. Literature review

Many scholars have proposed theories and built models to explain the exit decision under aggregate fluctuations, with notable examples including Clementi and Palazzo (2013) and Gomes (2001), as well as Lin and Wu (2003), Pieralli et al (2013), Murto and Terviö (2014), and Katchova and Ahearn (2017). In the context of the COVID -19 crisis, Crouzet and Tourre (2021) examine the effects of credit interventions on investment decisions in a partial equilibrium framework, while Miyakawa et al. (2021) study the effects using firm-level data for Japan and show heterogeneity in terms of exit rates across industries and regions. Additionally, Kalemli-Ozcan et al. (2020) use a cost-minimization model to measure the impact of the COVID-19 shock on business failures.

Despite the implementation of policy responses, the crisis has led to a serious threat of business continuity, resulting in an increase of firms leaving the market. Academic literature presents varying approaches in explaining firm bankruptcy or market exit, with economists suggesting either a lack of access to additional funds (Kalemli-Ozcan et al., 2020, Crouzet and Tourre; 2021) or an increase in leverage and the risk of "debt overhang" (Boot et al :2020). This paper will analyze these issues by examining the optimal decision to cease the business of a firm operating under persistent productivity uncertainty.

The issue of optimal capital structure and trade-off theory has been widely discussed in corporate finance, particularly in light of recent initiatives to implement credit support packages in order to sustain companies and avoid financial failure during the crisis. Titman and Tsyplakov (2007) analyze the ability of firms to adjust their capital structure choices during financial distress and find that they tend to increase their market debt ratios in the face of negative output shocks. Tserlukevich (2006) uses a real option model to explore financing behavior and suggests that, given transaction costs, debt is often the primary source of external financing for new investments. Hennessy and Whited (2005) also observe a negative relationship between profitability and debt and explain it as a “no anomaly” within the Q-theory. Bond et al (2010) use a model of debt policy in the presence of quadratic adjustment costs to demonstrate that the difference between the discount rate and the interest rate is a key factor in the decision to borrow. Finally, Eberly and Abel (2004) note that even if the effect size of adjustment cost on cash-flow is small, it can provide useful information about the capital stock growth prospects.

3. Conceptual framework of firm decision

Prior to delving into dynamic modelling based on the real options approach, we propose a conceptual framework that delineates the exit rules and their relationship to both capital accumulation and fluctuation in productivity. Building off the structural approach put forth by Olley and Pakes (1996), we present the logic of intertemporal investment and exit decisions. Unlike the study by Olley and Pakes (1996), our model takes into account a crisis context due to the COVID-19 outbreak, in which investors have to stop investing and only have access to debt as a source of capital. Our model also accounts for the impact of aggregate economic shocks on productivity. Furthermore, these shocks affect financing behavior and leverage adjustment. To provide a basic understanding of the exit decision, we start by assuming that capital accumulation does not incur any adjustment costs. We investigate a binary choice between staying in the market or not, in the context of a starting situation in which a firm is facing an unexpected liquidity shortfall due to the sudden outbreak of the COVID-19 pandemic and its resultant impacts on business activities. Kalemli-Ozcan et al. (2020) have demonstrated that liquidity shortfall is the primary cause of bankruptcies among small and medium enterprises. Liquidity shortfall occurs when the combination of firm revenue and internally available cash is unable to cover operational expenses, periodic financial obligations, or investment expenditures. Temporary liquidity shortfalls are typically caused by unexpected circumstances, such as production system failure or weakening aggregate demand, which lead to lower revenues within a given period. This scenario is reflective of the situation of a distressed firm during the COVID-19 pandemic, with a severely reduced demand and a heightened exposure to idiosyncratic risk across a variety of sectors.

In order to mitigate the effects of the crashing stock market, limited access to equity financing, and a sweeping lock-down, governments have been providing their respective economies with liquidity via loans and guarantees. It is assumed that these funds will be used by firms to replenish their capital stock (K), thus enabling them to remain operational. However, Bénassy-Quéré and Weder di Mauro (2020) suggest that the resulting debt overhang can lead to substantial economic costs but may be manageable in the post-pandemic era when firms can finance their operations without external support.

Usually, capital stock accumulation, K_{t+1}, is described by the following fundamental function:

$$K_{t+1} = (1-\rho)K_t + I_t,\ \ i \geq 0 \quad (1)$$

with I_t and $\rho \in [0,1]$ define respectively the needed investment and the depreciation rate of the capital. The no-investment situation combined to issuing new debt to be able the stay in business will also increase the capital stock from K_t to K_{t+1} during [t, t+1].

Furthermore, we assume that anyway the needed liquidity to stay in business will entirely be funded by debt:

$$I_t = D_t, \quad D_t \geq 0$$

As in Carvalho et al (2017), when capital stock is growing by debt, it can be described as:

$$K_{t+1} = (1-\delta)K_t + D_t$$

The firm decision dynamics proposed by Olley and Pakes (1996) as well as Jovanovic (1982), Clementi and Palazzo (2016) and Gomes (2001) suggest that firms maximize their expected discounted value of future revenue, under uncertain market conditions, by choosing whether to exit or remain in the market through investment in physical capital. Within this framework, it is assumed that under aggregate fluctuation, the decision maker has the ability to make endogenous decisions to invest or exit the market.

- Increase the leverage level through refilling capital stock by issuing new debt to stay in business.
- Quit the business irreversibly, repurchases all existing debt at its face value before selling the company and receive a sell-off value *£.* This decision can be explained by the fact that once production stopped, it will be very costly to restart under the pandemic uncertainty.

Hence, the decision at the beginning of each period can be formulated as maximization problem where decision maker takes financing action to maximize the firm´s net revenue:

$$\max_K(\pi_t, £)$$

The max operation means that the decision maker will compare the value of net revenue generated by staying in business and the sell-off value.

The decision depends on the fluctuation of net revenue. The net revenue per period is defined as, $\pi_t(s_t, q_t)$, a function of the perception of the future market structure defined by the state vector *s*, given the current information and the control decision vector *q*.

As in Olley and Pakes (1996), we assume that the decision to continue or not the business activity depends on a vector of states variable $s = (K, w) \in S$ the state space, where:

- w_t: a stochastic shock observed by the manager at each time *t*, and may be defined as the index of firm efficiency, profitability or productivity parameter and depends primarily on market condition.

- K_t: the firm´s capital stock at time *t*.

The productivity parameter *w* can be observed through an index assumed to be known for the firm and evolves stochastically over time according to a Markov process, where the conditional The productivity parameter w can be observed through an index assumed to be known for the firm and evolves stochastically over time according to a Markov process, where the conditional distribution of next period's profitability index w_{t+1} will be denoted as H (w_{t+1} | w_t). That means that decision maker must maximize the expected value of net revenue giving the perception of market interaction at time t. Since the decision can be taken when the decision maker is supposed to know the productivity level at beginning and the selling-off value is predetermined, the exit rule will be completely and simply defined by simple exit threshold.

For each capital stock level, there is an exit threshold productivity. If productivity evolves to reach a level below w the firm exit, otherwise, the firm will stay in operation. The decision problem has two control variables. The decision vector, denoted a, is given by:

- A binary control variable χ, where:

$$\chi_t = \begin{cases} 1 \;\; if\ the\ firm\ decide\ to\ continue\ at\ time\ t\ where\ w > w^* \\ 0 \;\; if\ the\ firm\ decide\ to\ abondon\ at\ time\ t\ where\ w \leq w^* \end{cases}$$

- A continuous control variable D_t, since the decision maker have control over firm´s financing policy.

$\chi_t = 1$ denotes that firm continue with staying in market and and $\chi_t = 0$ denotes a business's exit.

The productivity threshold is defined as:

$$w_t^* = w_t^*(K_t, D_t)$$

The decision maker chooses its debt level based on its beliefs about future productivity. The decision to borrow depends on its capital stock and productivity:

$$D_t = D_t(w_t, K_t)$$

The debt financing term implies that debt level increases in positive productivity shock. The firm which undergoes positive productivity shock in the period *t* will need to borrow more to cover increasing operation expenses. According to Olly and Pakes (1996), two decision rules $D_t(.)$ and $w_t(.)$, respectively defining the debt financing and exit decisions, are determined by a Markov-perfect Nash equilibrium. These decisions are contingent upon the parameters which specify the equilibrium and are contingent on the market efficiency of the decisions taken on time.

4. Framework of the dynamic programming

The decision to remain in business or to cease operations is dependent upon the assessment of future market conditions based on the available information. Dynamic programming offers the benefit of permitting the identification of optimal financing approaches in the presence of uncertain events, such as the occurrence of forced outages and major issues (Rothwell and Rust, 1995).

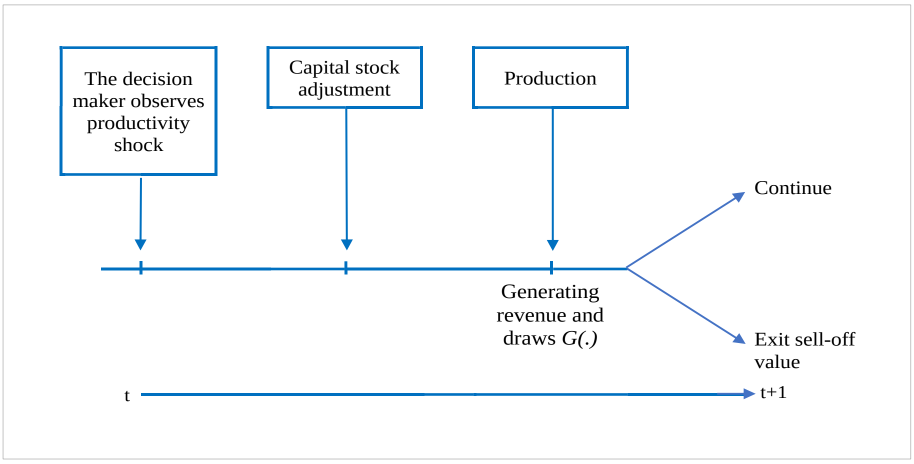
The DP model consists of:

- a discrete time index, $t \in \{0,1,2,\dots,T\}$
- a vector of state variables, s
- a control decision vector $a = (\chi, D)$
- a revenue function $\pi_t(s_t, q_t)$
- a discount factor, β
- a transition density (probability) $H(s_{t+1}|s_t)$

Since the purpose of our model is to define the exit decision as a function of debt financing strategy, we develop a tax neutral model and focus on instantaneous earnings before taxes and depreciation. In this case, π (·) is per-period revenue and G (·) is the payment occurring with the decision of staying in business, assuming that the decision to exit the market is a costless decision.

Figure 1

Timing of one-period events



Source. authors' contribution

According to our conceptual framework presented in section 2, we assume that at the beginning of period t-1, the decision maker observes productivity shocks before making the decision to continue operations through debt financing. The timing of one-period events between t-1 and t is described in Figure 1.

If the firm decides to continue $X_t = 1$ based on available information about the aggregate shocks at *t* the productivity should be higher to exit threshold: $w_t > w^*$, the continuation needs to be financed. The financing decision depends on the level of accumulated capital and the observed aggregate shocks at t.

The Bellman equation for the resulting mixed discrete-continuous control problem is given by:

$$V(w_t, k_t) = \max_X \left\{ \varphi, \sup_{l_t \geq 0} \pi_t(w_t, K_t) - G(D_t) + \beta \int E[V_{t+1}(w_{t+1}, K_{t+1})H(w_{t+1}|w)] \right\} \quad (2)$$

Similarly, Winter (1998) used the dynamic approach to study Firm's joint investment and exit decisions as mixed discrete-continuous dynamic problem. The author used Euler equation with applying some technical assumption and particularly a bounded return for unobserved efficiency index. Using Winter (1998) method based on Euler equation, within the framework of our analysis, gives the following results:

$$\frac{dG(D)}{dD} = \beta \int X_{t+1} \left\{ \frac{d\pi(s_{t+1})}{dK} + (1-\delta) \frac{dG\big(D(s_{t+1})\big)}{dD} \right\} H(ds_{t+1}|s) \qquad (3)$$

where s is the vector of state variables, s= (*k,w)*

In Winter (2012), it is accepted that the construction of closed-form solutions disregards essential financial principles such as the exposure to financial constraints and the nature of cash flows. In order to explain exit decisions beyond the intricacy of firm dynamics and structural model estimation, a real option approach is utilized in the subsequent sections to suggest a precise analytical resolution of the exit problem for an individual business. Following the same rationale of the firm dynamics, a straightforward stochastic model is instituted. In this basic application, the exit decision is studied as a function of capital accumulation, incorporating modifications in debt structure and productivity random shocks which are reflective of random shocks affecting the market structure in the context of the COVID-19 pandemic.

4. Exit real option model

The key concept of this section is to explain the exit decision using the real option approach proposed by Dixit (1993) and Dixit and Pindyck (1994). We present a straightforward reduced form model in which the revenue generated from business activity is a function of productivity that fluctuates randomly in time. By allowing for capital adjustment, the model investigates the relationship between the abandonment point and debt policy. We formulate the exit decision for a company facing two frictions: a convex quadratic debt adjustment cost and a sell-off value, indicating that the decision-maker may also choose to abandon the business in order to limit losses, even when continuing operations would be economically advantageous. Without these financial frictions, the firm can accumulate negative profits indefinitely, which renders the exit option valueless. To keep the model as simple as possible, we make the following assumptions: (1) debt is the only available external financing option, and the firm will be able to reissue new debt without any additional costs such as agency costs or other transaction costs; (2) the firm has no savings or internal cash or liquidity reserves available to finance its business activities; and (3) no tax shields will be generated by debt interest payment.

The Asset-to-Debt-to-Capital Ratio remains the sole source of growth of the Capital Stock, with Borrowing more by Firms motivated by the need to remain in business and ensure profitable business activity in accordance with Stockholders' requirements. In response to the COVID-19 Crisis, Fiscal Stimulus Policies assume that Financing Decisions are Tax Neutral, thereby rejecting the traditional Trade-Off Theory. This assumption reflects the changes in Financial and Tax Systems resulting from Financing Behaviour and Government Measures. This framework allows for the analysis of the effect of the Leverage Ratio on the Stopping Point.

To solve this Stopping Problem, a Dynamic Programming Approach is used, which consists of two steps: Step 1, assuming that the Value Function is known; and Step 2, solving the Bellman Equation in order to find the Exit Trigger.

We consider a single existing firm active with *K* units of capital stock. Business activity of the firm yields an instantaneous revenue:

$$Y_t(K) = w_t K \qquad (4)$$

with w_t the productivity parameter that could also reflect profitability and market efficiency.

For simplification reasons, our revenue function omits labor and instead focuses on capital. We denote by K_0 the initial investment made by the firm to enter the market. The firm faces stochastic market conditions where w_t follows a geometric Brownian process with drift and variance parameter μ and σ *:*

$$dw_t = \mu w dt + \sigma w_t dz_t \qquad (5)$$

where dz_t is the increment of a standardized Wiener process (i.e., with mean $E(dz) = 0$ and variance $E(dz^2) = dt$).

Modelling operating revenue as a geometric Brownian motion implies that the current operating revenue is known for a given initial productivity level but future revenues are unknown and are log-normally distributed with a variance that increases with the given time horizon.

Our model assume that capital stock is non-stochastic and "quasi-fixed". At the same time, we assume that capacity may be optimized by allowing change in *K* through debt financing associated with adjustment costs determined by the needed loan D and the capital stock level for the same period. Based on a large literature related to Q-theory, we model H(.) as the function of debt adjustment cost as a quadratic function of debt-to-assets ratio $L = D/K$. The debt adjustment cost is the cost charged by the creditor when the firm need more debt, in the sense that H(.) allows the firm to grow its capital stock. H(.) is convex and increasing in L. The function of debt adjustment cost can be written as:

$$H(K)=\frac{\vartheta}{2}\left(\frac{D}{K}\right)^2 K = \left(\frac{\vartheta}{2}L^2\right)K \qquad (6)$$

The parameter ϑ measures the cost of additional borrowing mainly interest without any additional costs.

Net revenue at any time t is given by:

$$\pi(w_t, K) = w_t K - \left(\frac{\vartheta}{2}L^2\right)K - C(K) \qquad (7)$$

where model $C(.)$ represents the total disbursement associated with capital stock variation $(\rho - n)$ *K,* where ρ*K* is the depreciation of the capital stock, while nK is the periodic amount of the new issued debt.

To avoid liquidity issues during the crisis, we assume that the firm don´t have to repay contracted debt. $C(K)$is defined as:

$$C(K) = (\rho - n)K$$

We are interested in a critical threshold for the stochastic productivity w^*that triggers the market exit. Exit is irreversible and generates a liquidation value φ without an additional exit cost. w^* represents the boundary between the continuation and the exit region.

$$\chi(w) = \begin{cases} 0 \; if \; w_t \le w^* & continue \\ 1 \quad othewise & Exit \end{cases}$$

The decision problem constitutes an optimal stopping problem that has two state variables the current productivity level and a discrete variable that indicates whether the operation is active or $(\chi = 1)$ or not $(\chi = 0)$. The decision problem can be solved by stochastic dynamic programming.

The objective of the decision maker is to maximize the expected presented value of net profit $\pi(w_t, K)$, where the future cash-flows are discounted at rate δ, with $\delta - \mu > 0$. The convexity of adjustment cost implies that a higher debt level yields a higher debt adjustment cost that constitutes loss of a fraction of revenue. When firms are highly levered under crisis, it will be too risky to continue business activity with costly debt financing. Capital adjustment through debt financing decisions will be constrained here by the opportunity to earn a liquidation value by exiting the market and selling the firm. The optimal exit policy depends both on revenue and capital stock initial level but also the liquidation value.

As evidenced by Pieralli et al. (2013), in contrast to Dixit (1989) and Dixit and Pindyck (1994), our model does not consider combined entry and exit decisions simultaneously; instead, it focuses on the optimal timing of the exit decision. The difference between these approaches and our model lies in the specification of the profit value function.

Utilizing dynamic programming, we define the value function $V(w_t)$, which represents the value of the expected discounted future cash flows for a current productivity level. Later, we calculate the option to exit for liquidation value φ. With an infinite time horizon, and with fixed initial capital stock K_0, the value of an active firm depends on w. Given the time increment $dt,$ the value of the firm $V(w_t, K)$ or simply $V(w)$ at a certain time t is equal to the sum of the net revenue and expected capital gain over (t; t + dt):

$$\delta V(w)dt = \pi(w)dt + EdV(w)$$

Applying Ito Lemma yields the following familiar partial differential equation (EDP):

$$\frac{1}{2}\sigma^2 w^2 V''(w) + \mu w V'(w) - \delta V(w) + \pi(w) = 0$$

By assuming the linearity of production function, the general solution to this equation is represented as:

$$V(w) = A_1 w^{\beta_1} + A_2 w^{\beta_2} + \left(\frac{wK_0}{\delta-\mu} - \frac{\left(\frac{\vartheta}{2}L^2\right)K_0}{\delta} - \frac{C(K_0)}{\delta}\right), \text{ if } w > w^*$$

The term between parentheses represents the expected present value of the net revenue generated by keeping the firm in the market forever and come from investing initial capital stock K_0 to enter the market. The value of the exit option is given by the first two terms where A_1 and A_2 are two constants to be determined and $\beta_1$and $\beta_2$are respectively the negative and positive roots of the fundamental quadratic equations (see Dixit and Pindyck (1994), with:

$$\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left[\frac{\mu^2}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2\delta}{\sigma^2}} < 0$$

and,

$$\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left[\frac{\mu^2}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2\delta}{\sigma^2}} > 0$$

The general solution shows that a decision maker will wait until the value of the firm is lower than the liquidation value B to get out of the market. The value of the exit option will be worthless if productivity is high for the constant A_2 associated with the positive root need to be 0 (zero). The general solution becomes:

$$V(w) = \begin{cases} A_1 w^{\beta_1} + \left(\frac{wK_0}{\delta - \mu} - \frac{\left(\frac{\vartheta}{2}L^2\right)K_0}{\delta} - \frac{C(K_0)}{\delta}\right), if\ w > w^* \\ \varphi \qquad , \quad if\ w \leq w* \end{cases}$$

Constant A_1 and the threshold w^* must be determined by the boundary conditions. Thus, the solution of EDP can be obtain by imposing the value matching and smooth pasting condition, at the stopping trigger w^*, we obtain the following equations:

$$V(w^*) = B$$

$$V'(w^*) = 0$$

The conditions above yield:

$$A_1 = -\frac{w^{*1-\beta_1} K_0}{\beta_1(\delta - \mu)}$$

$$w^* = \varphi \frac{\beta_1}{\beta_1 - 1} \frac{(\delta - \mu)\left[\frac{\vartheta}{2}L^2 + (n - \rho)\right]}{\delta}$$

The analytical solution indicates the productivity level at which the firm would optimally exit. According to Dixit (1989), this exit trigger w* implies "how bad things can get" before a business will be abandoned and where the decision maker knows that one can never restart it later.

5. Numerical results

We can now use numerical simulation of the stochastic productivity evolution to illustrate the impact of productivity uncertainty σ and leverage level L variation on the analytical solution of exit trigger. The base case parameters listed in Table 1 are used.

Table 1

Base case parameters

Parameters	
ϑ	10%
σ	30
μ	0
δ	6%
w_0	100%
K_0	100
D	50
L	50%
Depreciation rate ρ	10%
Debt reissuance rate n	10%
Liquidation value	50

Source. authors' calculation

The impact of productivity uncertainty is captured by the multiple sell-off value $\frac{\beta_1}{\beta_1 - 1}$, which is lower than unity. The multiple of the selling-value φ decreases in σ (see table 1). This implies, as expected, the exit trigger clearly decreases as the uncertainty increases. Higher variance makes the profitability risk higher and the trigger to exit lower. Table 1 shows the variation the option´s multiple as a function of σ (20%, 30% and 50%), for $\mu = 0$ and $\delta = 6\%$.

Table 2

Multiplier sensitivity to uncertainty

	$\boldsymbol{\sigma = 20\%}$	$\boldsymbol{\sigma = 30\%}$	$\boldsymbol{\sigma = 50\%}$
Beta 1	-1.303	-0.758	-0.354
Multiple of $\boldsymbol{\varphi}$	0.566	0.431	0.262

Source. authors' calculation

Figure 2

Productivity sample paths for σ=50%, 20%

Source. authors'

An increase in the volatility of firm productivity implies that is no longer profitable to stay in the market.

Figure 3

Exit trigger

Source. authors'

Thus, it is surprising that the exit trigger does not depend directly on the initial capital stock level, despite the fact that the continuation payoff is obviously greater for firms with larger capital stocks. W* increases with leverage ratio, implying that firms with higher leverage ratios have a higher exit trigger compared to less leveraged ones. Our model therefore explains not only firms' financing behavior but also their decision to exit. Specifically, when the firm is in the region of optimal continuation, its leverage ratio increases in response to a negative productivity shock. Furthermore, for a given level of capital stock, firms tend to issue new debt to improve their survival chances. The threshold function also reveals that, for a given depreciation rate of the capital stock, w increases with the number of debt units. Consequently, successive units of debt require successively higher thresholds of productivity, which contradicts the theoretical inverse relationship between productivity and leverage.

6. Concluding remarks

In this paper, we developed a dynamic programming model to study the optimal stopping timing in the presence of stochastic productivity. Our model includes debt adjustment cost in the determination of exiters behavior. Inspired by Olley and Pakes (1996) one of the earliest treatments of exit with aggregate fluctuations, we assumed that exit decision is subjected to productivity uncertainty. To analyze how productivity uncertainty and leverage level jointly affect the exit threshold, we used real options as a natural framework to explain analytically decision regularities in a crisis context. Our extended exit option model, that incorporates debt adjustment cost function, allows us to explain the effectiveness of generous credit policy with the aim of supporting firms to face financial shortfall during the COVID-19 crisis. The framework of the analysis violates the tradeoff theory assumption, which is the tax benefit of debt financing.

The COVID-19 crisis has had a severe impact on firm liquidity, leading to an abrupt financial shortfall and a large wave of exits across markets. In order to protect both employment and firms, governments have implemented credit support programs with flexible terms to provide access to liquidity during the crisis, prompting questions about the relationship between exit decision and a highly leveraged economy. This paper aims to analyze the effect of an increasing leverage level on the decision to exit under productivity uncertainty.

To this end, we developed a dynamic programming model to investigate the optimal stopping timing in the presence of stochastic productivity. Our model considers debt adjustment costs in the determination of exit behavior, and is motivated by Olley and Pakes (1996), one of the earliest treatments of exit with aggregate fluctuations. We assume that exit decisions are subject to productivity uncertainty and use real options to explain analytically the decision regularities in a crisis context. Our extended exit option model, which incorporates a debt adjustment cost function, allows us to explain the effectiveness of generous credit policy in supporting firms to face financial shortfalls during the COVID-19 crisis. The framework of the analysis challenges the tradeoff theory assumption, which states that debt financing has tax benefits.

Our analysis reveals that, as anticipated, uncertainty acts as a motivating factor for firms to leave the market. Specifically, higher volatility reduces the exit threshold and decreases the chances of a firm's survival. However, the exit threshold is an increasing function of leverage ratio for a given initial capital stock. Thus, credit intervention policy remains effective in the crisis situation, not only by providing liquidity, but also by increasing incentives to stay in the market. Our general modelling framework can be extended to take into account the heterogeneity of the COVID-19 effect on productivity by using industry data instead of modelling it as a standard stochastic model. The model can also be extended by incorporating a Cobb-Douglas technology specification, comprising other input factors that can influence the financing decision. Nonetheless, our model is simple and comprehensive enough to comprehend the exit behaviour in a complex crisis context.

The data that support the findings of this study are openly available in the following references.

References

1. Bartik, A.W., Bertrand, M., Cullen, Z., Glaeser, E.L., Luca, M. and Stanton, C.T. (2020). The impact of COVID-19 on small business outcomes and expectations", Proceedings of the National Academy of Sciences 117(30).
2. Becker, B., Hege, U. and Mella-Barral, P. (2020), Corporate debt burdens threaten economic recovery after COVID-19: Planning for debt restructuring should start now, VOX, CEPR Policy Portal.
3. Bénassy-Quéré, A. and Weder di Mauro, B. (2020), Europe in the time of COVID-19: A new crash test and a new opportunity. In: "Europe in the time of Covid-19", Bénassy-Quéré, A and B Weder di Mauro (eds.). VoxEU.org eBook, CEPR Press.
4. Bond, S., Soderbom, M. and Wu, G.L. (2010). The Effects of Financing Constraints on Firm Investment: A Structural Econometric Analysis. Available at: https://www3.ntu.edu.sg/home/guiying.wu/BSW_fc_201002.pdf.
5. Boot, A., Carletti, E., Kotz, H.H., Kranhen, J.P., Pelizzon, L. and Subrahmanyam, M.G., (2020), Coronavirus and financial stability 3.0: Try equity-risk sharing for companies, large and small. In: "Europe in the time of Covid-19", Bénassy-Quéré, A and B Weder di Mauro (eds.). VoxEU.org eBook, CEPR Press.
6. Carvalho, L., Lima, G.T. and Serra, G. (2017). Debt-financed knowledge capital accumulation, capacity utilization and economic growth, *FEA-USP, Department of Economics Working Paper*, 2017-32.
7. Clementi, G.L. and Palazzo, B. (2013). Entry, exit, firm dynamics, and aggregate fluctuations, *NBER Working Paper* 19217.
8. Crouzet, N., Tourre, F. (2021). Can the cure kill the patient? Corporate credit interventions and debt overhang. Available at SSRN: https://ssrn.com/abstract=3954581 or http://dx.doi.org/10.2139/ssrn.3954581
9. Dixit, A. (1989). Entry and Exit Decisions under Uncertainty. *Journal of Political Economy*, 97(3), 620–638.

10. Dixit, R.K. and Pindyck R.S. (1994). Investment under uncertainty. Princeton University Press.
11. Eberly, J.C. and Abel, A.B. (2004), Q Theory Without Adjustment Costs & Cash Flow Effects Without Financing Constraints. *Society for Economic Dynamics, 2004 Meeting Papers*, 205.
12. Gomes, J. (2001). Financing investment. *American Economic Review*, 91(5), 1263-1285.
13. Hennessy, C.A. and Whited, T.M. (2005). Debt Dynamics, *The Journal of Finance*, 60(3), 1129-1165.
14. Jovanovic, B. (1982). Selection and the Evolution of Industry. *Econometrica*, 50(3), 649-670.
15. Kalemli-Ozcan, S., Gourinchas, P.-O., Penciakova, V. and Sander, N. (2020). COVID-19 and SME Failures. *IMF Working Paper*, 2020/207.
16. Katchova, A.L. and Ahearn, M.C. (2017). Farm entry and exit from US agriculture. *Agricultural Finance Review*, 77(1), 50-63.
17. Lin, C.T. and Wu, C.R. (2003), Batch process and transfer decisions in foreign market: A real options model. *Applied Stochastic Models in Business and Industry*, 19(2), 121-131.
18. Megginson, W.L. and Fotak, V. (2020). Government Equity Investments in Coronavirus Bailouts: Why, How, When?. Available at: http://dx.doi.org/10.2139/ssrn.3561282
19. Miyakawa, D., Oikawa, K. and Ueda, K. (2021). Firm exit during the covid-19 pandemic: Evidence from Japan, *Journal of the Japanese and International Economies*, 59, 101118.
20. Murto, P. and Terviö, M. (2014). Exit options and dividend policy under liquidity constraints. *International Economic Review*, 55(1), 197-221.
21. Olley, S. and Pakes, A. (1996). The Dynamics of Productivity in the Telecommunications Industry. *Econometrica*, 64(6), 1263-1297.
22. Pieralli, S., Huettel, S. and Odening, M. (2013). A model of firm exit under inefficiency and uncertainty. *SiAg-Working Paper*, 14. https://edoc.hu-berlin.de/handle/18452/3884
23. Rothwell, G. and Rust, J. (1995). Optimal response to a shift in regulatory regime: The case of the US nuclear power industry. *Journal of Applied Econometrics*, 10, S75–S118.
24. Sinagl, P. (2020). When does Cash-flow Risk Matter to Investors? Evidence from the COVID-19 Pandemic. Available at SSRN: https://ssrn.com/abstract=3566511 or http://dx.doi.org/10.2139/ssrn.3566511
25. Titman, S. and Tsyplakov, S. (2007). A Dynamic Model of Optimal Capital Structure, *Review of Finance*, 11(3), 401–451, https://doi.org/10.1093/rof/rfm017

26. Tserlukevich, Y. (2006), Can Real Options Explain Financing Behavior?. Available at: http://dx.doi.org/10.2139/ssrn.889447
27. Winter, J. (1998, and 2012). Investment and Exit Decisions at the Plant Level: A Dynamic Programming Approach. Springer Edition.