

CONSTANT CORRELATION MODEL FOR OPTIMAL PORTFOLIO FORMATION AND EXPECTED SHORTFALL RISK MEASUREMENT: EMPIRICAL EVIDENCE FROM INDONESIAN STOCK MARKET

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Abstract

Stock investment is one option of investment choice with risks. Investors can reduce their risk by combining several stocks and then forming a portfolio. One method to form an optimal portfolio is by using the Constant Correlation Model (CCM) method. The CCM method focuses on the correlation between stocks and the Excess Return to Standard Deviation (ERS) value. Calculation of risk in the portfolio can use the Expected Shortfall (ES) method. ES is defined as a loss with a value exceeding VaR. ES is considered appropriate for measuring portfolio risk compared to VaR because it fulfils the subadditivity property. The subadditivity shows the advantage of portfolio formation. The object of this research is to form an optimal portfolio using the CCM method and ES risk measure on the Indonesian Stock Market Indices, that is IDX30 index. The daily return of IDX30 Index is analysed for the period January 2021 - December 2022. The formed portfolio contains 3 stocks, namely BMRI with a weight of 46.263%, KLBF of 39.255%, and MDKA of 14.482%. The Expected Shortfall value at a trust level of 95% is 5.408% for the next week.

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Keywords: cut off rate, excess return to standard deviation, IDX30, Monte-Carlo simulation, optimum weight

JEL Classification: G11; G32

1. Introduction

Investments are a form of planning to anticipate future circumstances. Tandelilin (2010) explained that investments are an activity of using current money to gain profits in the future. Those who make investments are known as investors. Investors can invest in various things, such as stocks. A stock can be described as an investment document that shows ownership of the company that released it (Adnyana, 2020). BEI (2023) noted that Indonesia has about 42 types of stock indexes. The stock index can help investors in investing. The purpose of a stock index is to show the movement of stock prices that fit certain characteristics. One of the indexes that Indonesia has is the IDX30.

Investors can diversify their funds into several stocks at once to minimize the possibility of risk. Stock diversification is done by combining several stocks into a portfolio. A portfolio is a set of investment instruments that are arranged to achieve an investment purpose (Priyadi, Wijaya, & Ready, 2021). A stock portfolio can be formed using the Constant Correlation Model (CCM) method. The CCM method focuses on the correlation between stocks. It uses the Excess Return to Standard Deviation (ERS) value to form a portfolio.

Generally, risk of a portfolio is determined by calculating the Value at Risk (VaR). However, VaR has a shortcoming to be considered appropriate in measuring portfolio risk, which is the subadditivity. The subadditivity reflects that there are benefits of portfolio formation. The Expected Shortfall (ES) method is used as an alternative for the shortage of VaR. Typically, Expected Shortfall is defined as a risk that has a value that exceeds the Value at Risk.

The research of Saepudin, Yasin, and Santoso (2017) talked about risk analysis with VaR and ES on a single stock. The result of the research showed that ES has a higher value than VaR. Furthermore, research by Nugroho and Nurcahyo (2019) talked about portfolio formation with the Constant Correlation Model on the Bisnis27 index. From that research, there are four stocks that form the portfolio. Then, research by Salsabilla, Maruddani, and Rusgiyono (2023) talked about Mean-Value at Risk optimization model to form an optimal

portfolio on stocks listed in IDX30 index. From that research, there are three stocks that form the portfolio. In this research, the Constant Correlation Model method is used to obtain the optimal stock portfolio and calculation of the risk by Expected Shortfall on stocks listed in IDX30 index. This method is used to form an optimal portfolio as a consideration in investing funds along with the possible risk that may occur.

2. Literature Review

2.1 Stock Characteristics

Return is the result of doing an investment (Maruddani & Purbowati, 2009). Calculation of return can use the equation (1) (Jorion, 2007).

$$R_{i(t)} = \ln \left[\frac{P_{(t)}}{P_{(t-1)}} \right] \quad (1)$$

with $R_{i(t)}$ is the return of the i -th stock in period t , $P_{(t)}$ is the closing price at time t , and $P_{(t-1)}$ is the closing price at time $(t-1)$.

According to Tandellilin (2010), the expected return is the average of the return value. Calculation of expected return can use the equation (2).

$$E(R_i) = \frac{\sum_{t=1}^n R_{i(t)}}{n} \quad (2)$$

with $E(R_i)$ is the expected return of the i -th stock, $R_{i(t)}$ is the return of the i -th stock in period t , and n is the number of stock periods.

The investor not only considers the return on the investment but also its risk. Risk is the difference that may occur between actual return received and expected return (Tandellilin, 2010). Calculation of risk can use the equation (3).

$$s_i = \sqrt{\frac{\sum_{t=1}^n (R_{i(t)} - E(R_i))^2}{n - 1}} \quad (3)$$

with s_i is the standard deviation of the i -th stock, $R_{i(t)}$ is return of the i -th stock in period t , $E(R_i)$ is the expected return of the i -th stock, and n is the number of stock periods.

For the CCM method, the return of stock data is normally distributed. In general, the Kolmogorov-Smirnov test can be used to ensure that the data is normally distributed. The Kolmogorov-Smirnov test is a hypothesis testing method to test the distribution suitability, such as the normal distribution (Oppong & Agbedra, 2016). The hypotheses are as follows:

H₀: Data is normally distributed.

H₁: Data is not normally distributed.

Significance level: α

Test statistics:

$$D = \sup_x |F(x) - F_0(x)|$$

Test criteria:

H₀ rejected if $D > D^*(\alpha)$ atau p-value $< \alpha$. $D^*(\alpha)$ is the critical value obtained from the Kolmogorov-Smirnov table.

A multivariate normality test is used to test whether two or more variables jointly are normally distributed. According to Johnson and Wichern (2007), this test is performed with the Mahalanobis distance value. According to Maruddani (2019), Kolmogorov-Smirnov test can be used for multivariate normality testing. The hypotheses are as follows:

H₀: Data is normally distributed multivariate

H₁: Data is not normally distributed multivariate

Significance level: α

Test statistics:

$$D = \sup_{d_t^2} |S(d_t^2) - \chi_p^2|$$

Test criteria:

H₀ rejected if $D > D^*(\alpha)$ atau p-value $< \alpha$. $D^*(\alpha)$ is the critical value obtained from the Kolmogorov-Smirnov table.

2.2 Constant Correlation Model (CCM) Portfolio

In 1987, Elton and Gruber started to develop the Constant Correlation Model method to form a portfolio. For the CCM method, Elton and Gruber (2014) state that each pair of stocks has the same correlation. According to Elton and Gruber (2014), on the Constant

Correlation Model method, all stocks are ranked based on Excess Return to Standard Deviation (ERS). The ranking is done starting from the stock with the largest to the smallest ERS value. Furthermore, stocks with negative ERS value are excluded from the portfolio formation candidates. The calculation of stock ERS can use the equation (4).

$$ERS_i = \frac{(E(R_i) - R_f)}{s_i} \quad (4)$$

with ERS_i is ERS of the i -th stock, $E(R_i)$ is expected return of the i -th stock, R_f is risk free rate, and s_i is standard deviation of the i -th stock.

Portfolio formation with CCM will contain stocks with high ERS. Determining a high ERS value requires a constraint called the Cut-off Rate (C^*). The C^* value is obtained through the correlation coefficient value. The correlation coefficient (r) is used to determine how much the relationship between two variables is. The correlation coefficient is between -1 and 1. The calculation of the correlation coefficient can use Pearson Product Moment with the equation (5) (Sugiyono, 2015).

$$r_{ij} = \frac{n \sum_{t=1}^n R_{i(t)} R_{j(t)} - (\sum_{t=1}^n R_{i(t)}) (\sum_{t=1}^n R_{j(t)})}{\left[\sqrt{n \sum_{t=1}^n R_{i(t)}^2 - (\sum_{t=1}^n R_{i(t)})^2} \right] \left[\sqrt{n \sum_{t=1}^n R_{j(t)}^2 - (\sum_{t=1}^n R_{j(t)})^2} \right]} \quad (5)$$

with r_{ij} is correlation coefficient of stock i with j for $i \neq j$, n is number of stock periods, $R_{i(t)}$ is return of the i -th stock in period t , and $R_{j(t)}$ is return of the j -th stock in period t .

The calculation of the constant correlation value which is the average of the stock returns correlations can use the equation (6) (Nugroho & Nurcahyo, 2019).

$$r = \frac{\sum_{i=1}^N \sum_{j=1}^N r_{ij}}{\left(\frac{N(N-1)}{2} \right)} \quad (6)$$

with r is constant correlation, r_{ij} is correlation coefficient of stock i with j for $i \neq j$, and N is number of stocks.

After obtaining the constant correlation value, the Cut-off (C_i) value of the stock is calculated to determine the C^* value. The calculation of C_i value is using the equation (7).

$$C_i = \frac{r}{1 - r + (ir)} \sum_{i=1}^N \frac{(E(R_i) - R_f)}{s_i} \quad (7)$$

with C_i is cut-off value of the i -th stock, r is constant correlation, i is the order of stocks from the highest to the lowest ERS, $E(R_i)$ is expected return of the i -th stock, R_f is risk free rate, and s_i is standard deviation of the i -th stock.

The C^* value is the C_i value where the last ERS value is still greater than the C_i value or determined based on the highest C_i value. If $ERS_i > C^*$, the stock will be included in the optimal portfolio.

According to Elton and Gruber (2014), the weight of each stock that forms the optimal portfolio with the Constant Correlation Model method can be calculated using the equation (8).

$$w_i = \frac{z_i}{\sum_{i=1}^N z_i} \quad (8)$$

with:

$$z_i = \frac{1}{(1 - r)s_i} \left(\frac{(E(R_i) - R_f)}{s_i} - C^* \right) \quad (9)$$

w_i is weight of the i -th stock, r is constant correlation, $E(R_i)$ is expected return of the i -th stock, R_f is risk free rate, and s_i is standard deviation of the i -th stock.

According to Tsay (2002), portfolio return calculation is using the equation (10).

$$R_{p(t)} = \sum_{i=1}^N (w_i \cdot R_{i(t)}) \quad (10)$$

The calculation of portfolio expected return is with the equation (11).

$$E(R_p) = \sum_{i=1}^N (w_i \cdot E(R_i)) \quad (11)$$

2.3 Portfolio Risk

In 1977, Boyle introduced a method to measure risk called Monte Carlo simulation. The calculation of VaR with Monte Carlo simulation assumes returns are normally distributed. In addition, this simulation can also be used to determine losses that exceed VaR.

According to Alijoyo, Wijaya, and Jacob (2019), the number of repetitions for Monte Carlo simulation can be determined using the error value (ε) with the equation (12).

$$K = \left(\frac{3\sigma}{\varepsilon}\right)^2 \quad (12)$$

with ε is error value, σ is the standard deviation of all stocks, and K is the number of repetitions.

Calculate the standard deviation (σ) value can use the equation (13).

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (X_i - \mu)^2}{N}} \quad (13)$$

with X_i is sum of the i-th stock returns, μ is average return of all stocks, and N is the number of stocks.

Next is to determine the acceptable absolute error value. Absolute error value $\leq \alpha$ then it can be calculated with the equation (14).

$$\varepsilon = \frac{\mu}{\left(\frac{1}{\alpha}\right)} \quad (14)$$

The calculation steps of VaR and ES using Monte Carlo on the portfolio are as follows:

1. Determine the parameter values for the variables (in this case, stock returns). Portfolio-forming stock returns are assumed to follow a multivariate normal distribution, so the parameters needed are the average portfolio-forming stock returns and the covariance variance matrix.
2. Simulate the return values by randomly generating multivariate normally distributed stock returns with the parameters obtained in step (1).
3. The return value of each stock at time t ($R_{i(t)}$) which is generated in step (2) is used to calculate the expected return and standard deviation of the portfolio.
4. Calculate the $(1-\alpha)$ quantile value of the portfolio return obtained in step (3), denoted by R^* .

5. Calculate the VaR value at the confidence level $(1-\alpha)$ in time t and initial investment of V_0 .
6. Calculate the ES value at the confidence level $(1-\alpha)$ in time t and initial investment of V_0 .
7. Repeat step (2) to step (6) for K times so that K values of VaR and ES will be obtained.
8. Calculate the average of VaR and ES at the confidence level $(1-\alpha)$ obtained from step (7) to stabilize the VaR and ES values because the VaR and ES values generated by each simulation are different.

Value at Risk (VaR) is the maximum loss obtained with a certain confidence level and over a certain period of time when the market is stable (Jorion, 2007). The calculation of VaR at the confidence level $(1-\alpha)$ in the time period t and the initial capital of V_0 can use the equation (15).

$$VaR_{(1-\alpha)}(t) = V_0 R^* \sqrt{t} \quad (15)$$

Value at Risk has a shortcoming to be considered appropriate in determining portfolio risk, namely the subadditivity. The subadditivity shows that the risk in a portfolio with two assets will be smaller or equal to the risk of each asset. Expected Shortfall (ES) comes as an alternative for that shortcoming. According to Denuit, Dhaene, Goovaerts, and Kaas (2005), ES satisfies the subadditivity property to measure portfolio risk. The subadditivity reflects the fact that there should be benefits of diversification. Diversifying a loss will reduce the risk. Each risk random variable X_1 and X_2 applies:

$$ES(X_1 + X_2) \leq ES(X_1) + ES(X_2) \quad (16)$$

According to Rahmawati, Rusgiyono, Hoyyi, and Maruddani (2019), Expected Shortfall (ES) at the confidence level $(1-\alpha)$ in the time period t and the initial capital of V_0 can use the equation (17):

$$ES_{(1-\alpha)}(t) = V_0 \left(\mu + \sigma \frac{\phi(VaR_{(1-\alpha)})}{\alpha} \right) \sqrt{t} \quad (17)$$

3. Methods

The data is collected from Yahoo Finance, the Bank of Indonesia, and the Indonesia Stock Exchange. The variables used are

the weekly closing prices of 21 stocks that are consistently listed on IDX30 for the period January 01, 2021 - December 30, 2022, and the Indonesian interest rate for the period January 2021 - December 2022. The analysis of data was done with Microsoft Excel and R software.

The steps in the analysis are as follows:

1. Input weekly closing prices data for 21 stocks that are consistently listed on IDX30 for the period January 01, 2021 – December 30, 2022, and SBI data for the period January 2021 – December 2022.
2. Calculate the return, expected return, and standard deviation of each stock.
3. Select stocks with positive expected return values, normally distributed returns, and low correlation.
4. Calculate the R_f and ERS value. Then sort the ERS value of each stock from largest to smallest and exclude stocks with negative ERS value.
5. Calculate the average value of stock returns correlation.
6. Calculate the C_i value of each stock and determine the *Cut Off Rate* (C^*) value.
7. Determine the candidate stocks that will form the portfolio with a value of $ERS > C^*$.
8. Calculate the z_i value of each stock. Then calculate the total of all z_i value.
9. Calculate the weight value of each stock that form the portfolio.
10. Perform multivariate normality test on the return data of stocks that form the portfolio.
11. Simulate the return of portfolio forming stock by generating random numbers.
12. Calculate the expected return and standard deviation of the portfolio.
13. Calculate the quantile value $(1-\alpha)$ of the portfolio return.
14. Calculate the VaR value at confidence level $(1-\alpha)$ in time t and initial investment of V_0 .
15. Calculate the ES value at confidence level $(1-\alpha)$ in time t and initial investment of V_0 .
16. Repeat step (11) to step (15) for K times.
17. Calculate the average of VaR and ES at the confidence level $(1-\alpha)$ to stabilize the values.

4. Results and Discussions

In this research, the highest expected return value is owned by ADRO stock of 0.00969 with a standard deviation of 0.05839. The lowest expected return value is owned by SMGR stock of -0.00602 with a standard deviation of 0.04501. The selected stocks are those with a positive expected return value. Based on the calculation results with the R program, it is known that 13 stocks have a positive expected return value, namely ADRO, BBCA, BBNI, BBRI, BMRI, ICBP, KLBF, MDKA, PGAS, PTBA, TBIG, TLKM, and TOWR. The overall expected return and standard deviation of each stock can be seen in Table 1.

Table 1

Expected return and standard deviation value of stocks

Stock Code	$E(R_i)$	s_i
ADRO	0.00969	0.05839
ANTM	-0.00260	0.07085
ASII	-0.00061	0.04051
BBCA	0.00197	0.02628
BBNI	0.00348	0.04274
BBRI	0.00138	0.03527
BMRI	0.00411	0.03469
CPIN	-0.00124	0.03946
ICBP	0.00049	0.02738
INDF	-0.00007	0.02691
INKP	-0.00259	0.06039
KLBF	0.00329	0.03156
MDKA	0.00428	0.05681
PGAS	0.00110	0.05365
PTBA	0.00259	0.04692
SMGR	-0.00602	0.04501
TBIG	0.00337	0.05118
TLKM	0.00097	0.02956
TOWR	0.00131	0.03901
UNTR	-0.00026	0.04663
UNVR	-0.00410	0.05434

Source: R programming output

The normality test is performed on stocks that have a positive expected return value. Based on the processing with R obtained at the significance level $\alpha = 5\%$, the stock returns of ADRO, BBCA, BBNI, BBRI, BMRI, ICBP, KLBF, MDKA, PGAS, PTBA, TLKM, and TOWR accept H_0 because these stocks have a p-value $> \alpha$ (0.05) which means that the stocks are normally distributed. Meanwhile, TBIG stock returns are not normally distributed because of the p-value $< \alpha$ (0.05).

Based on the calculation of R, the weekly risk-free rate (R_f) value is 0.0007232 or 0.07232%. The ERS value of each stock is shown in Table 2. Next, the ERS value of each stock is sorted from highest to lowest. Stocks that have a negative ERS value are excluded as candidates to form a portfolio.

Table 2
ERS value of stocks

Stock	ERS
BMRI	0.09754
ICBP	-0.00840
KLBF	0.08120
MDKA	0.06260
PGAS	0.00699
TLKM	0.00837
TOWR	0.01502

Source: R programming output

Table 3
Calculation of cut-off stocks

Ranking	Stock	ERS	C_i
1	BMRI	0.09754	0.01330
2	KLBF	0.08120	0.02145
3	MDKA	0.06260	0.02586
4	TOWR	0.01502	0.02481
5	TLKM	0.00837	0.02336
6	PGAS	0.00699	0.02203

Source: R programming output

Based on Table 2, the highest ERS value is owned by BMRI stock. After the ranking, the calculation of the cut-off for each stock and cut-off rate is done. Based on the calculation of R, the value of constant

correlation is 0.13635. Next, the cut-off (C_i) of each stock is calculated. The (C_i) value of each stock can be seen in Table 3.

Based on Table 3, it is known that the highest C_i value to be used as the C^* value is owned by MDKA stock of 0.02586. The stocks with ERS value smaller than C^* will be excluded as the candidate to form the portfolio. The comparison between ERS and C^* can be seen in Table 4. Based on Table 4, it is known that there are three stocks with ERS value greater than C^* and are used as portfolio formers, namely BMRI, KLBF, and MDKA.

Table 4
The Comparison between ERS with cut-off rate

Stock	ERS		C^*
BMRI	0.09754	>	0.02586
KLBF	0.08120	>	0.02586
MDKA	0.06260	>	0.02586
TOWR	0.01502	<	0.02586
TLKM	0.00837	<	0.02586
PGAS	0.00699	<	0.02586

Source: R programming output

Based on the calculation of R, the weight (w_i) value for each stock can be seen in Table 5. Table 5 shows that the weight for each stock is 46.263% of the initial investment capital for BMRI stock, 39.255% of the initial investment capital for KLBF stock, and 14.482% of the initial investment capital for MDKA stock.

Table 5
Weight value of stocks

Stock	z_i	w_i
BMRI	2.39246	0.46263
KLBF	2.03004	0.39255
MDKA	0.74894	0.14482

Source: R programming output

A multivariate normality test was conducted on BMRI, KLBF, and MDKA stock return with the Kolmogorov Smirnov test. Based on the calculation of R, the p-value (0.493) > α (0.05) is obtained. At a significance level of $\alpha = 5\%$, it can be concluded that the stocks return that form the portfolio are normally distributed multivariate.

The calculation of VaR and ES value can be done with Monte Carlo simulation. Based on the calculation of R, the number of repetitions that will be used in the calculation of VaR and ES in this research is 45 times and then the average value of VaR and ES is calculated. At a confidence level of 95% and a period of one week with the calculation of R, it is obtained the value of VaR = 0.04621 and ES = 0.05408. Based on that result, it shows that at the 95% confidence level, the VaR value is 4.621% while the ES is 5.408% for the next week. The ES value is greater than the VaR value, so it means that the loss that will occur exceeds the VaR value by 5.408% for the next week. If an investor with an initial capital of 100,000,000.00 IDR, the loss that the investor will receive in the next week is 4,621,000.00 IDR and there is still a possibility of a loss exceeding this value which is 5,408,000.00 IDR. The ES value is more appropriate than the VaR value in the context of a portfolio so that the maximum loss that will be received is 5,408,000.00 IDR for the next week prediction.

This research forms an optimal portfolio of IDX30 stocks using the Constant Correlation Model (CCM) in the latest Indonesian stock market condition and estimates its future risk using Expected Shortfall (ES). It contributes to the empirical literature on portfolio formation by providing a practical guide for investors to form optimal portfolios. In the research of Nugroho and Nurcahyo (2019) who formed a portfolio with the Constant Correlation Model on the Bisnis27 index produced 4 portfolio-forming stocks, namely BBKA, AKRA, PTBA, and BDMN. In this research, the Constant Correlation Model optimal portfolio formation is carried out on a different index, namely IDX30 and produced 3 forming stocks, namely BMRI, KLBF, and MDKA. Research by Salsabilla et al. (2023) formed an optimal portfolio with Mean-VaR optimization applied to 23 stocks that were consistently listed on the IDX30 for the 2020 period and produced three forming stocks, namely CPIN, ERAA, and TLKM. Based on that research, it is also obtained that the VaR value of the portfolio is 3.992% for the next day. In this research, portfolio formation applied to 21 stocks listed on the IDX30 for the 2021 – 2022 period and obtained a portfolio VaR value of 4.621% for the next week. Research by Saepudin et al. (2017) shows that the ES value of Unilever Indonesia stock at 3.942% is greater than the VaR value at 3.425% for the next day. In this research, the ES value of the portfolio at 5.408% is greater than the VaR value at 4.621% for the next week. In the concept of risk, especially portfolios,

it is more appropriate to use the Expected Shortfall value to estimate the loss that will occur because it fulfills the subadditivity.

5. Conclusion

The purpose of the research is to form an optimal portfolio of IDX30 stocks using the latest data and the Constant Correlation Model (CCM). The research then uses Monte Carlo simulation to predict the future risk of the portfolio using Expected Shortfall (ES). Portfolio formation using the Constant Correlation Model method applied to stocks that were consistently listed on the IDX30 for the period January 2021 - December 2022 produces 3 optimal portfolio-forming stocks, namely BMRI, KLBF, and MDKA. The amount of weight for each stock forming the optimal portfolio with the Constant Correlation Model method is for BMRI stock by 46.263%, KLBF stock by 39.255%, and MDKA stock by 14.482%. The calculation of risk value in the Constant Correlation Model optimal portfolio with Expected Shortfall at the 95% confidence level is obtained at 5.408% of the initial capital for the next week. This shows that the loss that will occur exceeds the VaR value of 4.621% for the next week.

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