OPTIMAL HEDGE RATIO IN TURKISH STOCK INDEX FUTURES MARKET: A DECO-FIAPARCH APPROACH

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Abstract

This paper adopts a new approach called DECO-FIAPARCH model for estimating the optimal hedge ratio (HR) in Turkish Stock Index Futures market in the presence of asymmetry and long memory. The study covers the period from May 3, 2005 until April 4, 2019, total of 3,508 daily observations. The DECO-FIAPARCH model shows that, on average, a $1 long position in the spot market can be hedged for $0.95316 with a short position in the futures market. Furthermore, optimal hedge ratio is time-varying and takes value between 0.52258 and 1.5263. This demonstrates that investors should revise their positions actively by considering the fluctuating cross correlations in spot and futures markets.

Keywords: Time-Varying Hedge Ratio; Asymmetry**;** Long Memory; Fractional APARCH

JEL Classification: G10**;** G11**;** G13

1. Introduction

It is an undeniable fact that risk management is gaining more importance than ever before in the recent years. The growing interdependence of financial markets forces the investors who want to invest in portfolios of assets to face considerable amount of risk which has never been experienced before. As a hedge for the risk of price

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changes in the spot markets, many investors have started to follow stock index futures more closely. Through hedging, in other words investing in the spot market and taking an opposite position in the futures market at the same time, the investors try to minimize their risk. The crucial step of this hedging mechanism is to determine the optimal hedge ratio, which is essentially obtained from the coefficient of the regression between the change in the stock prices and the change in the hedging instruments (Hatemi-J. and Roca, 2006). However, the main question remains as to how many hedging instruments to use to manage stock price fluctuations in an optimal way. In this context, many studies have investigated how to find the best hedging strategy.

This study aims to investigate the hedging effectiveness of the Turkish derivative market with a dynamic model considering that information shocks, which may change depending on time and are eliminated at a hyperbolic rate, and have an asymmetrical structure. Although there is extensive literature on the hedging efficiency of the Turkish derivatives market using models such as OLS, VECM, GARCH, the number of studies that consider both conditional correlation and long memory in the calculation of the optimal hedge ratio is very few. The ARCH effect and long memory features in financial asset returns make it difficult to calculate a robust hedge ratio with models such as OLS, VECM, and GARCH. For this reason, this study will fill this gap in the literature by considering both conditional correlation and long memory in the calculation of the optimal hedge ratio.

In the following parts of the study, the literature review on optimal hedge ratio calculation, research methodology, results, and conclusion will be given, respectively.

2. Literature review

The literature on optimal hedging is generally separated into two groups: static methods and dynamic methods. Static methods suggest that the hedge ratio is fixed and not dependent on time which makes the calculation of a single hedge ratio sufficient. The first example of this kind of strategy is applied by Ederington (1979). He employed OLS model to estimate the hedge ratio minimizing risk. His results prove that the OLS method performs better than one-to-one hedge ratio for reduction of variance. Ederington (1979)'s OLS method has been followed by many authors as it is easy to implement e.g., Hill and Schneeweis, 1982; Figlewski, 1984; Toevs and Jacob, 1986; Benet, 1992. Nevertheless, in the OLS method, the cointegration between the spot market and futures markets is not taken into account. Moreover, it ignores the fact that the financial variables frequently display a unit root and time-varying variance-covariance structure which in turn result in model misspecification (for details please refer to Engle, 1982; Engel and Granger, 1987).

In order to overcome the deficiencies of OLS method, Ghosh and Clayton (1996), Lien and Tse (1999), Yang (2001), Floros and Vougas (2004), Lee et al. (2010), Degiannakis and Floros (2010), and Kostika and Markellos (2013) employed error correction model (ECM). ECM was observed to generate better results in nonstationary and cointegrated time series. By taking the long term cointegration term into account, Floros and Vougas (2004), Yang and Allen (2004), Bhaduri and Durai (2008), Degiannakis and Floros (2010), Prashad (2011) calculated the hedge ratio with vector error correction model (VECM).

As the static methods mentioned above focus only on minimizing the risk while calculating the hedge ratio, they disregard the time-varying change in the price and also overlook its effect on expected returns (Cecchetti et al., 1988). Therefore, Baillie and Myers (1991), Park and Switzer (1995), Haigh and Holt (2002), Rossi and Zucca (2002), Tse and Tsui (2002), Yang and Allen (2004), Degiannakis and Floros (2010), Prashad (2011), Gok (2016), Basher and Sadorsky (2016), Kharbanda and Singh (2018) and many others used models such as GARCH, BGARCH, VEC-GARCH, CCC-GARCH or DCC-GARCH to capture the dynamic nature of the prices. Although it is not easy to compute hedge ratio due to their complex algorithms, these dynamic models are generally shown to outperform the static models by taking conditional heteroskedasticity into account.

Compared to the models mentioned above, Engle and Kelly (2012) introduced a relatively new model called Dynamic Equicorrelation GARCH (DECO GARCH). DECO GARCH model is an advanced case of DCC model of Engle (2002) and cDCC model of Aielli (2013). Our empirical analysis adopts the Fractionally Integrated Asymmetric Power (FIAPARCH) model combined with the DECO. The study uses the FIAPARCH model rather than GARCH type models because of two reasons. First of all, the FIGARCH model counts in the long memory in the volatility of the return series (Hammoudeh et al., 2016). Although GARCH type models assume that a shock to financial time series disappear rapidly, fractionally integrated models suggest that shocks exposed by financial time series does not decline to zero exponentially and the decline is hyperbolically. The second reason is the asymmetry effect. GARCH type models have an important restriction of considering positive and negative shocks as equally important. Yet, it has been discussed that the volatility caused by a negative shock is expected to be higher than a positive shock of a similar size (Brooks, 2014).

To the best of our knowledge, this paper is the first to employ Dynamic Equicorrelation (DECO)-Fractionally Integrated Asymmetric Power ARCH (FIAPARCH) model for calculation of time-varying hedge ratio.

This paper is organized as follows. After the literature review presented in Section II, Section III includes the empirical method for DECO-FIAPARCH model. Section IV includes the empirical results of spot and futures markets. Section V reports and discusses the empirical results. Lastly, Section VI provides concluding remarks.

3. Methodology and data

3.1. Data

The series analyzed in the study are the daily spot and futures prices for the ISE-30 index. The data is taken from Bloomberg. Our sample period covers the period from first trading day of Turkish stock index futures market which is May 3, 2005 until April 4, 2019, summing up to 3,508 daily observations.

3.2. Model Specifications

Assume that for $t = 1, \dots, T, E_{t-1}[\varepsilon_t] = 0$ and $E_{t-1}[\varepsilon_t \varepsilon_t'] = H_t$, where $E_t[.]$ is the conditional expectation which uses the information set available at time t. The asset conditional variance-covariance matrix H_t is expressed as:

$$H_t = D_t R_t D_t \qquad (1)$$

where $R_t = [P_{ij,t}]$ is the conditional correlation matrix and the diagonal matrix of the asset conditional variances is given by $D_t = diag(h_{1,t}, \dots, h_{n,t})$.

Engle (2002) uses the right-hand side of Eq.(1) instead of H_t by putting forward the dynamic correlation structure called DCC.

$$R_t^{DCC} = (Q_t^*)^{-1/2} Q_t (Q_t^*)^{-1/2}, \tag{2}$$

$$Q_t^* = Diag[Q_t], \tag{3}$$

$$Q_t = (1 - \alpha - b)S + \alpha u_{t-1} u'_{t-1} + bQ_{t-1}, \tag{4}$$

where $u_{i,t}$ are the std. residuals, $S = [s_{i,j}] = E[u_t u'_t]$ is the nxn unconditional covariance matrix of u_t and a and b are no negative scalars fulfilling $a > 0$, $b \geq 0$, $a + b < 1$.

Within this context, Aielli (2013) demonstrates covariance matrix Q_t estimation in the method is unstable as $E[R_t] \neq E[Q_t]$ and recommends a model called cDCC which is consistent with the correlation-driving process.

$$Q_t = (1 - a - b)S^* + a\left(Q_{t-1}^{*1/2} u_{t-1} u'_{t-1} Q_{t-1}^{*1/2}\right) + bQ_{t-1} \tag{5}$$

where S^* is the conditional covariance matrix of $Q_t^{*1/2} u_t$

Engle and Kelly (2012) propose modeling ρ_t with the help of DCC model of Engle (2002) and its cDCC modification proposed by Aielli (2013) to create a conditional correlation matrix Q_t and after that taking the mean of its off-diagonal elements in order to lessen the estimation time by simplifying the procedure. This method is named as dynamic equicorrelation (DECO) model, and written as:

$$\rho_t^{DECO} = \frac{1}{n(n-1)} (\acute{J}_n R_t^{cDCC} J_n - n) = \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=k+t}^{n} \frac{q_{kl,t}}{\sqrt{q_{kk,t} q_{ll,t}}} \tag{6}$$

where $q_{kl,t}$ is the (k,l)th element of the matrix Q_t from the cDCC model.

Afterwards, conditional correlation matrix should be estimated. For that, following equicorrelation is implemented:

$$R_t = (1 - \rho_t) I_n + \rho_t J_n \tag{7}$$

where J_n is the nxn matrix and I_n is the identity matrix with n-dimension.

The presupposition of equicorrelation results in a less complex likelihood equation when ρ_t is acquired by Eq. (8):

$$L = -\frac{1}{T}\sum_{t=1}^{T}(\ln(1-\rho_t)^{n-1}(1 + (n-1)\rho_t))\frac{1}{1-\rho_t}\Big(\sum_{k=1}^{n}\varepsilon_{k,t}^2 - \frac{\rho_t}{1+(n-1)\rho_t}(\sum_{k=1}^{n}\varepsilon_{k,t}^2)\Big) \quad (8)$$

Baillie et al. (1996) suggested fractional integrated GARCH (FIGARCH) model for specifying long memory in return volatility. GARCH model is expressed as an ARMA (*m,p*) for squared error form

$$[1-\alpha(L)-\beta(L)\varepsilon_t^2] = \omega + [1-\beta(L)v_t] \quad (9)$$

where $v_t = \varepsilon_t^2 - \sigma_t^2$.

FIGARCH model results from standard GARCH model with fractional difference operator, $(1-L)^d$. Thus, FIGARCH model can be displayed as follows:

$$\emptyset(L)(1-L)^{\bar{d}}\,\varepsilon_t^2 = \omega + [1-\beta(L)]v_t \quad (10)$$

where $\beta(L)$ and $\emptyset(L)$ are the finite order lag polynomials with roots presumed to be placed outside of unit circle and d is the long memory parameter and $(1-L)^{\bar{d}}$ is the fractional differencing operator.

FIGARCH $(p,\bar{d},q)$ model turns into standard GARCH when $\bar{d}$ = 0 and IGARCH model when $\bar{d}$ = 1.

On the other hand, Tse (1998) claimed that the response o stock volatility to positive and negative shocks are asymmetrically and suggested the Fractionally Integrated Asymmetric Power (FIAPARCH) model. As negative shocks cause stock volatility to increase more compared to positive shocks, taking asymmetry effect into account along with long memory generates better results.

The Fractionally Integrated Asymmetric Power (FIAPARCH) (p,d,q) model is represented as follows:

$$h_t^{\delta/2} = \omega[1-\beta(L)^{-1} + [1-[1-\beta(L)]^{-1}\emptyset(L)(1-L)^d](|\varepsilon_t| - \lambda\varepsilon_t)^\delta \quad (11)$$

where $\omega, \beta, \emptyset$ and d are parameters that are needed to be determined.

The parameter d where $0 \leq d \leq 1$ tests the validity of long memory in the conditional volatility, δ stands for power term of returns for assumable structure in the volatility persistence, L represents the lag operator, and λ>0 denotes to the asymmetry parameter implying that stock volatility rises higher in negative shocks than positive shocks of similar size.

4. Results and discussion

First of all, the log returns are computed for spot and futures markets returns. Table 1 shows statistical properties related to the return series.

Table 1

Statistical Properties for Spot and Futures Markets Returns

	Spot Market Returns	Futures Market Return
Mean	0.000383	0.00039096
Maximum	0.12725	0.09657
Mininum	-0.10902	-0.097824
S.D.	0.017421	0.017415
S	-0.17253	-0.16795
K	3.4645	3.4504
Jarque-Bera	1771.8***	1756.6***
ADF	-33.6253***	-33.5158***
KPSS	0.0709367	0.0631605
Q (20)	35.3949***	31.6826***
Qs (20)	1231.91***	1319.60***
ARCH (20)	26.521***	28.187***

***Note**: S.D. refers to Standard Deviation, S and K are Skewness and Excess Kurtosis. Q (20) and Qs (20) are the empirical statistics for the LB test for spot and futures return autocorrelation and sqr. returns series, respectively. ADF refers to the unit root test and KPSS refers to the stationarity test. ARCH(20) test controls the ARCH effects. *** refer to the rejection of the hypothesis of "normality, homoscedasticity and no autocorrelation at the 1% significance level.*

It can be seen that average daily returns are positive throughout the sample period selected. The skewness results are negative for the whole return series along with that the spot and futures return series have very high excess kurtosis values. This result and the JB test statistics implies that the distribution function of spot and futures return series are leptokurtic and skewed. Thus, the null hypothesis which suggests "normality" is rejected. Additionally, the ADF unit root test suggested by Dickey and Fuller (1979), and the KPSS stationarity test suggested by Kwiatkowski et al. (1992) are implemented. These findings show that all return series are stationary. Moreover, so as to conclude for existence of ARCH effect, serial correlation in the residual term is examined. According to the results, there is a significant autocorrelation and existence of ARCH behavior in the entire markets just as supported by the Ljung-Box statistic. Thus, estimating a GARCH model specification is suitable for modeling situations including clustering volatility, fat tails and persistence for daily spot and futures market returns.

Figure 1 displays the time-variations for daily spot markets returns. Figure 2 plots the time-variation for daily futures market returns.

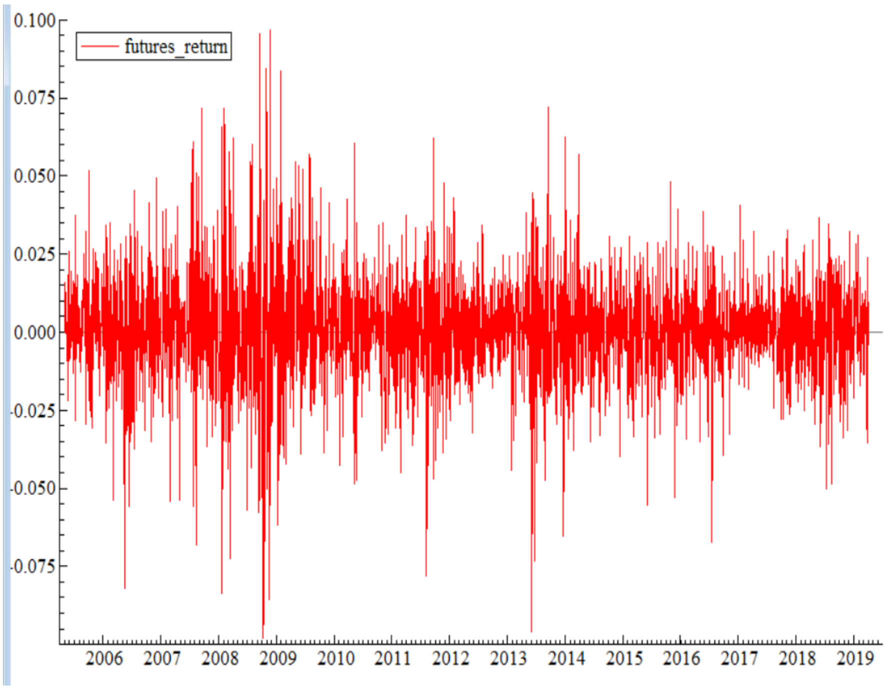
Figure 1

Time-Variations for Daily Spot Markets Returns

Source: Prepared by the authors

Figure 2

Time-Variations for Daily Futures Market Returns



Source: Prepared by the authors

In Table 2, the summary of test results for the DECO-FIAPARCH $(1, d, 1)$ model related to spot and futures returns are given. Panel A of Table 2 shows the results of FIAPARCH estimation for each return series. The fractional integrated coefficient (d) is significant for spot and futures returns implying volatility is strongly persistent. The long memory parameter "d" is higher in futures return series than in spot return series. In addition, $\lambda_{Assymmetry}$ term is positive and significant. It shows that compared to positive information, negative information shocks cause more volatility in the markets.

Table 2

DECO-FIAPARCH (1, d, 1) Test Results.

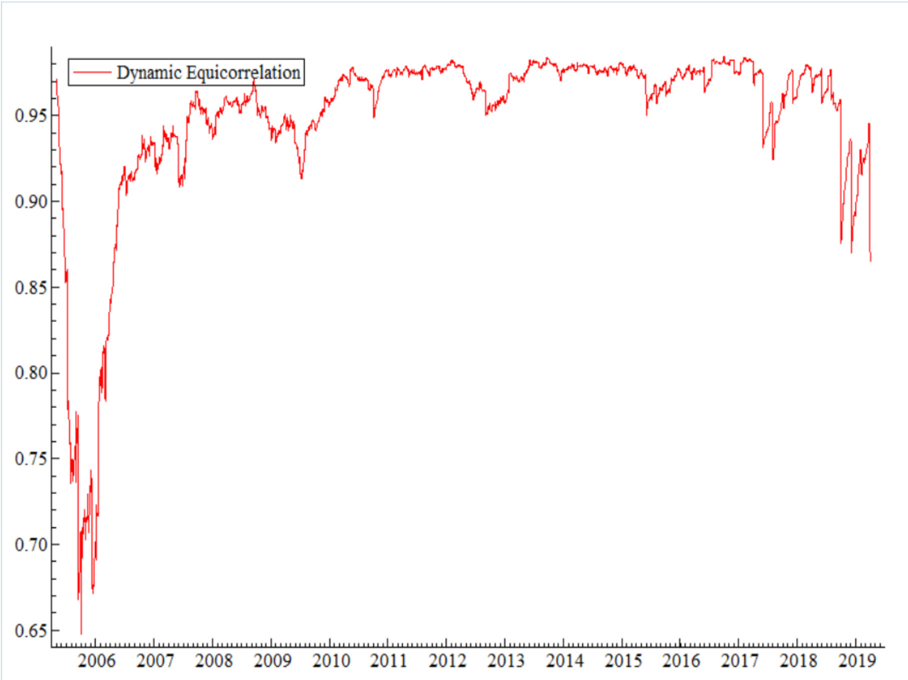
Panel A: Estimates of the univariate FIAPARCH model	**Spot Market Returns**	**Futures Market Returns**
Const. (m)	0.000609**	0.000723***
	(0.00025434)	(0.00025041)
Const. (v)	0.777214	1.250229
	(1.0104)	(2.1144)
d-Figarch	0.246559***	0.312659***
	(0.054628)	(0.072620)
$\emptyset_{Arch(1)}$	0.156081*	0.180324**
	(0.082281)	(0.080442)
$\beta_{Garch(1)}$	0.325997***	0.416435***
	(0.089125)	(0.11008)
$\lambda_{Assymmetry}$	0.420806***	0.336430***
	(0.13157)	(0.11659)
δ_{Power}	1.753216***	1.636502***
	(0.24188)	(0.33736)
Panel B: Results of the DECO model		
ρ_DECO	0.971277***	
	(0.0082268)	
α_DECO	0.016598***	
	(0.0039900)	
β_DECO	0.979541***	
	(0.0057932)	
Panel C: Diagnostic tests		
Qs (10)	6.89118	8.26222
	[0.7356766]	[0.6032400]
Qs (20)	11.9256	11.6461
	[0.9186128]	[0.9277194]

***Note**: Qs (10) refers to the L-B test statistics conducted to the sqr. std. residuals with 10 lags, Qs (20) refers to the L-B test statistics conducted to the sqr. std. residuals with 20 lags. *, ** and *** shows significance level at the 10%, 5% and 1%, respectively. The std. errors are given with"()" and p-values are given with "[ ]".*

In Table 2, Panel B section summarizes the estimates for the DECO process. The a_{DECO} coefficient is significant at the 1% level and positive, emphasizing that shocks between the futures and spot markets are substantial. In the whole cases, the β_{DECO} parameter is close to one and significant, verifying that volatility persistence is higher for spot and futures returns. Nevertheless, the dynamic equicorrelation is positive and close to one (0. 971277). This result indicates that hedging effectiveness is higher in the futures market than the spot market. In other words, futures market can hedge the risks in the spot market effectively. Given the diagnostic tests stated in Panel C in Table 2, considering the L-B test statistics for std. residuals and sqr. std. residuals, the null hypothesis which suggests no serial correlation is not rejected. This proves no indication of model misspecification.

Figure 3

Dynamic Equicorrelation for Spot and Futures Markets Returns



Source: Prepared by the authors

Figure 3 displays the dynamic equicorrelation for spot and futures markets returns as a group. It is clear from the figure that the correlations change in time throughout the sample period, implying that investors need to revise their positions in the futures market continuously for hedging the spot market risks.

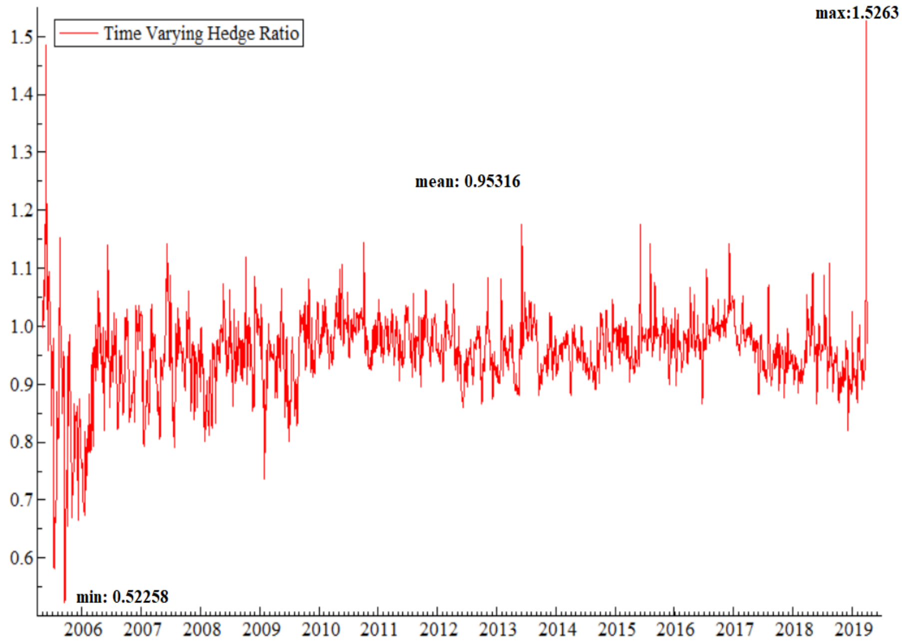
The aim of hedging is to minimize the risk of the price changes in the spot position with the usage of future contracts. The hedger should decide on the number of futures contracts to sell or buy for every unit of the spot asset. Referring to principles of the portfolio theory, Ederington (1979) and Figlewski (1984) define the problem of hedgers and calculate hedge ratios minimizing the variance of the portfolio. Optimal hedge ratio (HR) can be found by the following formula:

$$HR = \frac{Cov_{s\_f}}{Var_f}$$

where f is natural logarithms of futures and s stands for natural logarithms of spot prices.

Figure 4

Time-Varying Hedge Ratio



Source: Prepared by the authors

Figure 4 displays time-varying HR for the period selected. The mean value for the time-varying hedge ratio is 0.95316 which is significantly close to 1. This result suggest that the investors should take almost one-to-one long or short position in the futures market for hedging the risks occurring in spot market. On average, a $1 long position in the spot market can be hedged for $0.95 by taking a short position in the futures market. Surely, it is an expensive hedging opportunity. Also, time-varying hedge ratio ranges between 0.52258 and 1.5263. Therefore, it is significant that investors should update their positions dynamically by considering the changing cross correlations in spot and futures markets.

5. Conclusions

The expanding interdependence of financial markets urges investors to undergo serious risk than ever before. Still, investors have an option for risk reduction related to price fluctuations occurred in the spot market. By hedging, in other words investing in the stock index futures and spot markets simultaneously, investors are able to decrease the risk. However, what is crucial is the calculation of the optimal hedge ratio. Although various models aim to determine optimal hedge ratio, dynamic models are generally shown to outperform the static models as they regard conditional heteroskedasticity. Dynamic models emphasize that it is more appropriate to adopt a time-varying hedge ratio instead of a single and static ratio.

When the extensive literature on the subject is examined, it can be seen that many different methods are used to measure the hedge performance of futures markets. (see Ederington (1979), Benninga et al. (1984), Myers, Thompson (1989), Ghosh (1993), Ghosh and Clayton (1996), Lien and Tse (1999), Yang (2001), Moosa (2003), Harris, Shen, (2003), Choudhry, (2003), Kenourgios et al. (2008), Lee et al. (2009).

On the other hand, in studies taking into account that the correlation between spot and futures markets changes over time, it is claimed that the hedge ratio estimations of dynamic models are more robust than the static models (see. Floros and Vougas (2004), Ai et al. (2007), Degiannakis and Floros (2010), Celik (2014), Gok (2016), Buberkoku, (2019), Lai (2019)).

Although most of the recent studies have estimated hedge ratios with bivariate GARCH models, none of these models consider long memory in volatility. Bivariate GARCH models, among the dynamic models, assume that information shocks have a short-term effect on volatility. In this study, differing from the literature, the time-varying hedge ratio was calculated using fractional volatility models. This paper employs a new approach called DECO-FIAPARCH model in determining the optimal time-varying hedge ratio in Turkish Stock Index Futures market when asymmetry and long memory exist.

According to the research results, long memory (d) and leverage effect ($\lambda_{Assymmetr}$) have been determined in spot and futures ETFs. Negative information shocks exposed by both spot and futures markets cause more volatility in returns. In addition, information shocks that cause volatility are eliminated at a hyperbolic rate. On the multivariate GARCH analysis side, the conditional equicorrelation between the two ETFs is time-varying and approximately 97%. The persistence of the volatility spread between spot and futures ETFs is at 0.9795, indicating that it is highly persistent.

As a result of the calculations, the mean value of time-varying hedge ratios is found to be extremely close to 1 with a value of 0.95316. This mean value points out that people who want to invest in the Turkish spot market should take almost one-to-one buying or selling position in the futures market to be able to minimize the risk. Our results also demonstrate that time-varying hedge ratio changes between 0.52258 and 1.5263 for the sample period. So, it is critical for the investors to switch their positions actively by observing the fluctuant cross correlations in spot and futures markets.

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