LONGEVITY IMPACT ON THE LIFE ANNUITIES ON ROMANIA BY COMPARATIVE ANALYSIS WITH BULGARIA AND HUNGARY

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Abstract
People are living longer worldwide than they were two decades ago, as death rates are falling. There are significant differences on life expectancy between males and females, and across the countries, and between regions. But the real question is how will develop further the death rates and how much will increase the cost of life annuities or other retirement benefits. The present paper aims to analyse the development of the death rates on Romania, Bulgaria and Hungary, and how this trend will impact on the amount required to get a life annuity at the age of 65 years. Mortality and trend modelling is beyond the mathematical and econometric exercise, but this paper will look only on the methods for forecasting the mortality rates. This paper represents the authors' personal opinions and does not reflect the views of the institutions they are affiliated to.

Keywords: mortality rates, Lee-Carter method, Booth-Maindonald-Smith method, Coherent mortality forecasting

JEL Classification: J11, C53, G22

1. Introduction

In this paper, the impact of longevity on the life annuities will be study based on the evolution of mortality rates on Romania. The Lee-Carter method will be used to model the mortality pattern for Romania. The results will be compared with the results of applying the same method for Bulgaria and Hungary. For these countries the Lee-Carter model cannot be applied with satisfying results, since there have been major changes in mortality rates on last 25 years. But one of the strength of the model is that it really shows the

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predominant trend in the population, with a descending or ascending curve (Scherp, 2007).

However, the authors have not found any article where Lee-Carter model has been applied to mortality data from Romania. Further, we will focus more on the mortality trend on Romania and we will compare the results of Lee-Carter method with other methods Booth-Maindonald-Smith (BMS) (2002) and the Coherent functional demographic model as described in Hyndman, Booth & Yasmeen (2012).

The ageing of the population became a growing concern for governments and societies. The concerns are concentrated on the sustainability of pensions and health systems, especially given increased longevity. The government policy makers and planners are more interested nowadays in accurately modelling and forecasting age-specific mortality rates. A more accurate forecast mortality rates would be beneficial for decision makers with impact on the allocation of current and future resources. The forecast of mortality rates are of great interest to the insurance and pension industry, this industry being under the pressure of low interest environment also.

At least 10 new approaches for forecasting mortality rates and life expectancy using statistical modelling were proposed on the last decade by different authors. The method of Lee and Carter (1992) is still a milestone in demographic forecasting. "They used a principal component method to extract a single time varying index of the level of mortality rates, from which the forecasts are obtained using a random walk with drift. Since then, this method has been widely used for forecasting mortality rates in various countries" (Han, Heather, Hyndman, 2011).

The most important argument for Lee-Carter (LC) method is the simplicity and robustness of its application on the age-specific log mortality rates that have linear trends. On the other hand, the weakness of the LC method is that it attempts to capture the patterns of mortality rates using only one principal component.

Life expectancy at birth in early stages was about 20 to 30 years, by the middle of the 19th century it rose to 40 years. Rapid improvements began at the end of the 19^{th} century, so that by the middle of the 20th century it was approximately 60 to 65 years on the developed countries. At the beginning of the 21st century, life expectancy at birth reached about 70 years.

There are three stages on the development of the mortality rates. The first stage is characterized by no development of the life expectancy at birth and high volatility from one year to another. This is due to the fact that the people are less protected against extreme weather, flu epidemics. On this stage there is high mortality for young children also. Stage one can be compare with the situation of underdeveloped countries. On the second stage there is a steep increase of life expectancy and less volatility, as on the post industrial revolution period. This is due to medical developments, a better protection against weather conditions, improved hygiene, and cleaner drinking water. This stage is comparable with the situation of emerging countries. Stage three is typical for developed countries where the developments such as the quality of drinking water are reaching the limits. Particularly behaviour can cause more independency in development of the differences between male and female life expectancy (van Broekhoven, 2015). The progress on the medical field can improve the life expectancy on this stage, for example the use of angioplasty as a treatment in case of a heart attack had impacted the trend on 2001.

For life expectancy on the stages one and two it is easy to predict the future using the history but on the stage three the history can not really be used to predict the future. On the following decades, a decrease of life expectancy is possible, as well, in the coming 50 years. This could be caused by climate changes, development of resistance to antibiotics effects or behaviour causes, such as obesity.

Mortality and trend modelling is not just a mathematical and econometric exercise. Specially on the stage three, the history is a bad predictor of the future. Expert judgment method needs to be added, particularly from a medical/demographic view (van Broekhoven, 2015).

Periods and prospective life tables

There are two main types of life tables, period life tables and the prospective life.

The period life tables are tables with mortality rates, indicating the “probability” of dying during a one year period of time, depending on the age only. These tables take into account that mortality is on average greater when the person is older but they do not consider that mortality evolves with time. In this case, the future mortality is supposed to be exactly the same as the mortality today.

The prospective life tables, on the contrary, contain mortality rates depending on the age but also on the considered year. As mortality is evolving with time, expected future changes in the mortality rates can be captured with time-depending models. The prospective tables are also known as cohort life tables. In this present paper we will use prospective tables to calculate the actuarial present value of a life annuity at the age of 65.

2. Mortality forecasting methods

In this section, we review three methods for forecasting mortality rates and life expectancy, namely the LC method, the Booth-Maindonald-Smith method (BMS) and the new coherent functional demographic model as described in Hyndman, Booth & Yasmeen.

On the modelling process, it is necessary to transform the mortality rate data by taking the natural logarithm in order to avoid the high variance associated with high age-specific rates. The observed mortality rate at age x in year t is denoted $m_{t,F}$. This mortality rate is calculated as the number of deaths at age x in calendar year t, divided by the corresponding mid-year population aged x. The fitted and forecasts models are all in logarithmic scale.

2.1. Lee-Carter (LC) method

The structure of the model proposed by Lee and Carter (1992) is given by the equation:

$$ln(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t} \quad (1)$$

where,

a_x is the age pattern of the log mortality rates averaged across years;

b_x is the first principal component reflecting relative change in the log mortality rate at each age;

k_t is the first set of principal component scores by year t and measures the general level of the log mortality rates;

$\varepsilon_{x,t}$ is the residual at age x and year t.

The LC model in (1) could be regarded as over-parameterized in the sense that the model structure is invariant under the following transformations:

$$\{a_x, b_x, k_t\} \rightarrow \{a_x, b_x/c, ck_t\}$$

$$\{a_x, b_x, k_t\} \rightarrow \{a_x - cb_x, b_x, k_t + c\}$$

Lee and Carter (1992) imposed two constraints in order make the model identifiable, framed as:

$$\sum_{t=1}^{n} k_t = 0 \quad , \quad \sum_{x=x_0}^{x_p} b_x = 1 \quad (2)$$

where,

n is the number of years and p is the number of ages in the observed data set.

Further, on the LC method k_t is adjusted by refitting to the total number of deaths. The adjustment mentioned gives more weight to higher rates, thus roughly counterbalancing somehow the effect of using a log transformation of the mortality rates. The adjusted k_t is later extrapolated by the use of ARIMA models.

Lee and Carter (1992) used a random walk with drift model, which can be expressed by the equation:

$$k_t = k_{t-1} + d + e_t \qquad (3)$$

where,

d is the drift parameter and measures the average annual change in the series;

e_t is an uncorrelated error.

Random walk model with drift provides satisfactory results in many cases (Tuljapurkar, Li, and Boe 2000; Lee and Miller 2001. From the forecast of the principal component scores are obtained the forecast of the age-specific log mortality rates using the estimated effects on ages a_x and b_x in equation (1).

2.2. Booth-Maindonald-Smith (BMS) method

Booth-Maindonald-Smith (BMS) method is a variant of the LC method. BMS method has three differences from the LC method:

1. The fitting period is determined on the basis of a statistical ‘goodness of fit’ criterion, under the assumption that the principal component score k_t is linear.

2. The adjustment of k_t involves fitting to the age distribution of deaths rather than to the total number of deaths.

3. The jump-off rates are the fitted rates under this fitting regime (Han, Heather, Hyndman, 2011).

Under the LC method the best fitting time series model of the first principal component score is linear. And later Booth, Maindonald and Smith (2002) observed that the linear time series can be compromised by a structural change.

The BMS method, under the assumption of linear first principal component score, tries to achieve an optimal ‘goodness of fit’ by choosing the optimal fitting period. This optimal fitting period is selected from the possible fitting periods that are ending in year n and is determined based on the smallest ratio of the mean deviances of the fit of the underlying LC model to the overall linear fit.

Instead of fitting to the total number of deaths, the BMS method fits to the age distribution of deaths using the Poisson distribution to model deaths, and using deviance statistics to measure the ‘goodness of fit’ (Booth, Maindonald and Smith 2002). The jump-off rates are taken to be the fitted rates under this adjustment. (Han, Heather, Hyndman, 2011).

2.3. Coherent mortality forecasting: the product-ratio method with functional time series models

“Non-divergent forecasts for sub-populations within a larger population have been labelled “coherent” (Li & Lee 2005). Coherent forecasting seeks to ensure that the forecasts for related populations maintain certain structural relationships based on extensive historic observation and theoretical considerations. For example, male mortality has been observed to be consistently higher than female mortality at all ages, and while available evidence supports both biological/genetic and social/cultural/environmental/behavioural hypotheses as to why this is so, Kalben (2002) concludes that the determining factor is biological. Thus sex-differences in mortality can be expected to persist in the future and to remain within observed limits. Mortality forecasts for regions within the same country can also be expected not to diverge radically. Differences due to environmental and biological factors can be expected to remain unchanged, while differences due to social, political, behavioural and cultural factors are unlikely to produce persistent divergence in modern democracies where principles of equality apply. Similar arguments may apply to countries within a common economic and political framework such as the European Union.” (Hyndman, Booth, Yasmeen, 2012, p.3)

Up to the definition of coherence by Hyndman, Booth and Yasmeen from 2012, this has been defined the most precisely as the non-divergence of forecasts for sub-populations. They adopted a far more precise definition labelling “mortality forecasts as coherent when the forecast age-specific ratios of death rates for any two subpopulations converge to a set of appropriate constants.” (Hyndman, Booth, Yasmeen, 2012).

The coherent mortality forecasting is based on the method of forecasting the interpretable product and ratio functions of mortality rates under the functional data paradigm introduced in Hyndman & Ullah (2007).

„The product-ratio functional forecasting method can be applied to two or more sub-populations, incorporates convenient calculation of prediction intervals as well as point forecasts, and is suitable for use within a larger stochastic population modelling framework such as Hyndman & Booth (2008). The new method is simple to apply, flexible in its dynamics, and produces forecasts that are at least as accurate in overall terms as the comparable independent method." (Hyndman, Booth, Yasmeen, 2012).

Review of the coherent functional method

The method can be applied to any number of sub-populations of the main population by contrast to the common and well known approach of forecasting male and female age-specific death rates.

$m_{t,F}$ denotes the female death rate for age x and year t, t = 1,...,n. The logarithmic death rate will be modelled by the following:

$$y_{t,F}(x) = \log[m_{t,F}(x)]$$

Under the functional data paradigm, one assume that there is an underlying smooth function $f_{t,F}$ (x) that we are observing with error. Thus,

$$y_{t,F}(x_i) = \log[f_{t,F}(x_i)] + \sigma_{t,F}(x_i)\varepsilon_{t,F,i} \qquad (4)$$

Where, xi is the centre of age-group i (i = 1,...,p), $\varepsilon_{t,F,i}$ is an independent and identically distributed standard normal random variable and $\sigma_{t,F}(x_i)$ allows the amount of noise to vary with age x.

For smoothing, it is used the weighted penalized regression splines (Wood 1994) constrained so that each curve is monotonically increasing above age x = 65 (see Hyndman & Ullah 2007). The weights are to take care of the heterogeneity in death rates across ages. The observational variance $\sigma_{t,F}(x)$ is estimated by using a separate penalized regression spline of $\{y_t(x) - \log[f_{t,F}(x_i)]\}^2$ against x, for each t.

According with the Product-ratio method for males and females, the square roots of the products and ratios of the smoothed rates for males and females, are defined as follows:

$$p_t(x) = \sqrt{f_{t,M}(x) f_{t,F}(x)} \quad (5) \quad \text{and} \quad r_t(x) = \sqrt{f_{t,M}(x)/f_{t,F}(x)}$$

(6).

These functions will be modelled rather than the males and females death rates, with the advantage of that the product and ratio will behave almost independently of each other provided the sub-populations have approximately equal variances. Considered on the log-scale, these will transform in sums and differences which are approximately uncorrelated. In case of substantial differences in the variances of sub-populations, then the product and ratio will no longer be uncorrelated on the log scale. This can make the forecasts less efficient.

The functional time series models (Hyndman & Ullah 2007) for $p_t(x)$ and $r_t(x)$ are used:

$$\log[p_t(x)] = \mu_p(x) + \sum_{k=1}^{K} \beta_{t,k} \phi_k(x) + e_t(x) \quad (7)$$

$$\log[r_t(x)] = \mu_r(x) + \sum_{l=1}^{L} \gamma_{t,l} \Psi_l(x) + w_t(x) \quad (8).$$

Where, $\{\phi_k(x)\}$ and $\{\Psi_l(x)\}$ are the principal components obtained from decomposing $\{p_t(x)\}$ and $\{r_t(x)\}$ respectively, and $\beta_{t,k}$ and $\gamma_{t,l}$ are the corresponding principal component scores.

3. Empirical Studies

Fitting and applying the Lee-Carter model
The data available for the studied countries are:

Country	Time period	Ages used in the model	Source
Romania	1968-2013	0 – 100, 1 year age groups	EUROSTAT
Bulgaria	1947-2003	0 – 100, 1 year age groups	www.mortality.org
Hungary	1950-2001	0 – 98, 1 year age groups	www.mortality.org

The parameters are fitted using the method presented in Section 2.

We applied the Lee-Carter model to the data and start by analysing the $k_t$values and the b_x values. Residuals will then be studied. In case of data for Bulgaria and Hungary we have excluded the time periods before 1975 in order to improve the fit of the model. In case of data for Romania, the mortality rates were available on

Eurostat only until for ages between 0 and 86. Therefore, the value for 86 years was considered also for ages between 87 and 100.

Figure 1

LC parameters for Romania

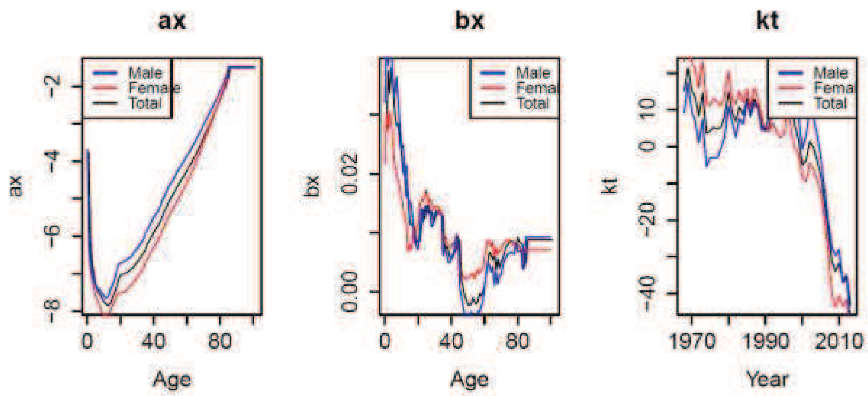


Figure 2

LC parameters for Bulgaria

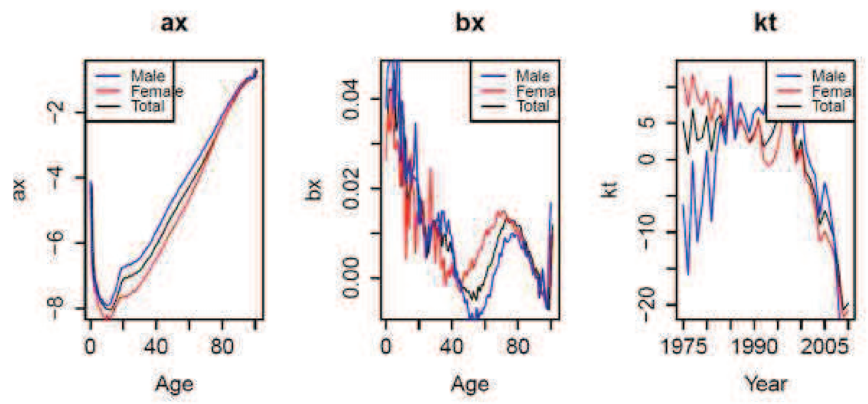
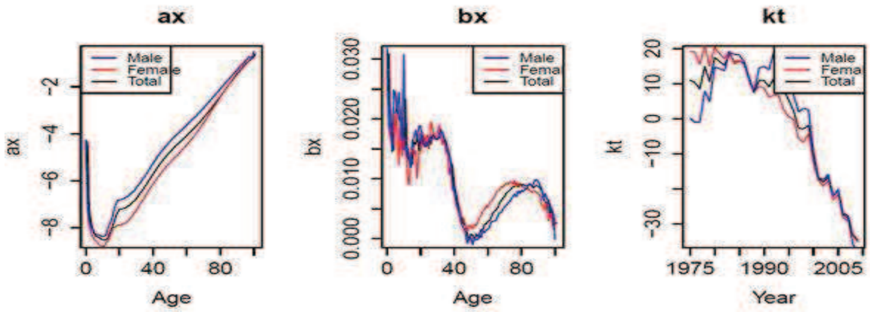


Figure 3

LC parameters for Hungary



The parameter a_x is reflecting the age pattern of the log mortality rates averaged across years. For the analysed countries the shape of a_x chart is very similar.

Regarding the parameter b_x which is reflecting relative change in the log mortality rate at each age, the most effected by the change in mortality rates is the youngest part of population and this is decreasing by age. But, there is a change in this trend for population at 20 years.

The less effected being the population between 45 and 65 years. Around the age of 80 there is a local maxim of b_x, and then is decreasing on the ages closing to 100.

Parameter k_t is giving the time trend of the general level of the log mortality rates. For Romania this estimated trend looks most similar for males and females comparing to the other two countries. In all three countries k_t for females has the general descending trend. By contrast k_t for males on Hungary and Bulgaria has an initial ascending trend till 1985.

In Bulgaria on 1993 the mortality rates start a new ascending trend till 1997 when are reversing again.

In case of Hungary, the mortality rates for males start increasing from 1988 till 1993. After 1993 the rates for males and female decreased till 2010.

For Romania, the mortality rates start to increase on 1990, specially the rates for males, till 1996. On 2001 there is another increase for 3 years. The development of the trend is shown in figure 4 and figure 5.

Figure 4

Parameter Kt estimated for mortality rate in case of Romania

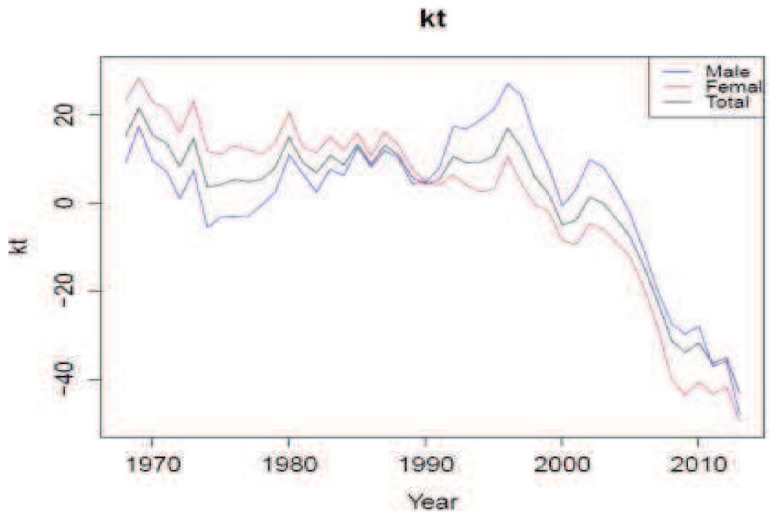
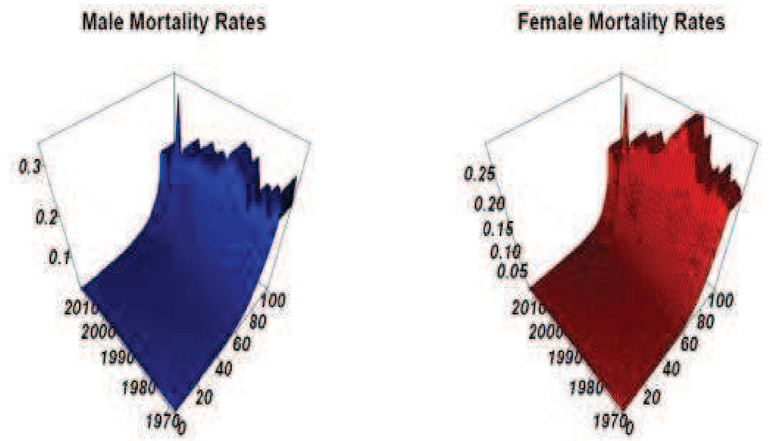


Figure 5

Mortality rates by age and year of birth on Romania



As conclusion, the parameter k_t shows the predominant trend in the mortality rates of the population, and this can be considered for forecasting the future development.

Based on the fitted parameter we forecasted the development of the mortality rates using an 80% confidence level.

Figure 6

Forecast of k_t parameter for Romania

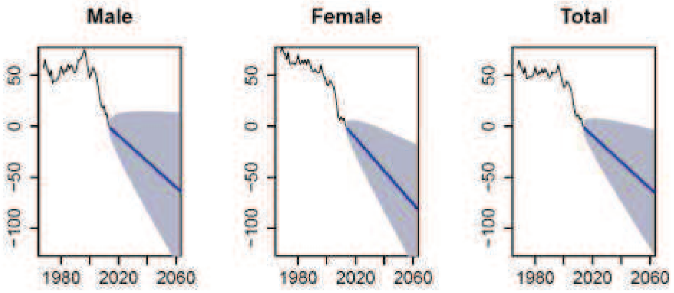


Figure 7

Forecast of k_t parameter for Bulgaria

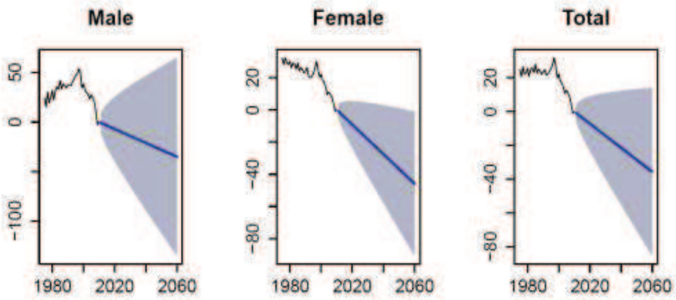


Figure 8

Forecast of k_t parameter for Hungary

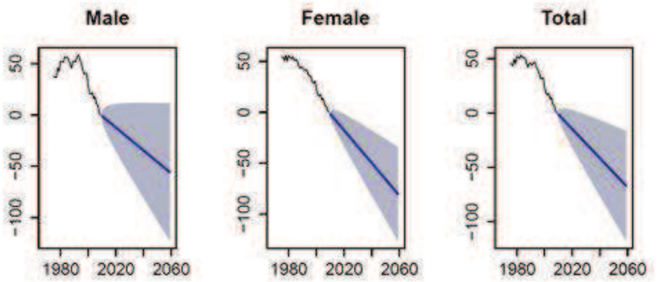
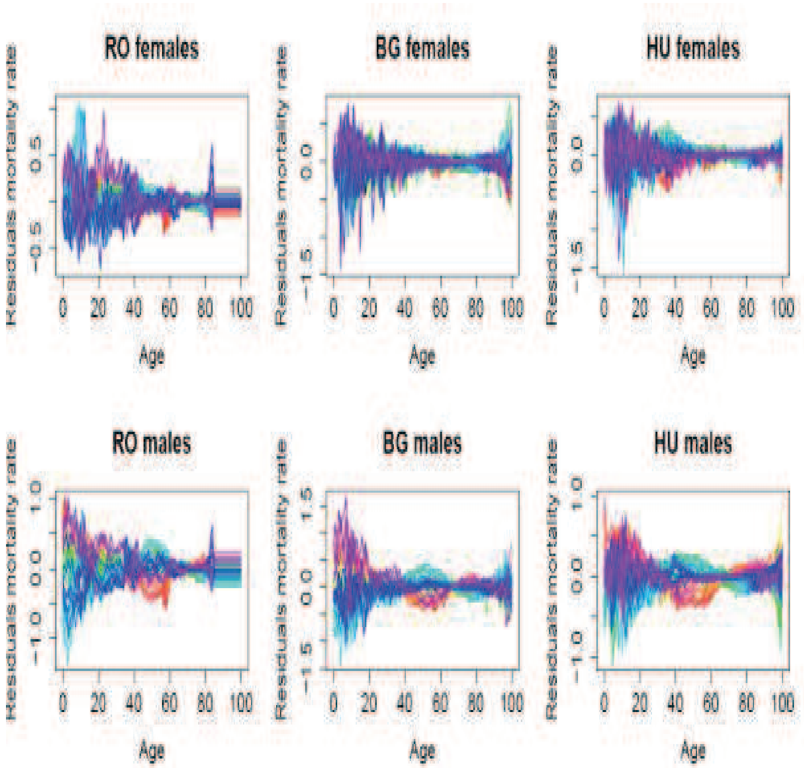


Figure 9

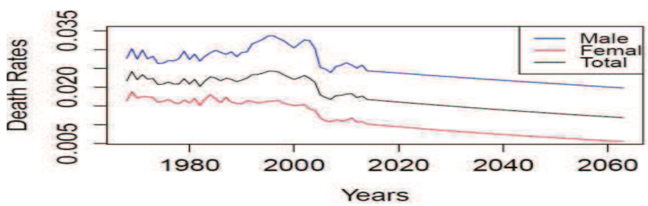
Residuals of LC parameter estimations by country



The distribution of the residuals across the ages is not uniformed and it shows that the model can not capture the change in case of the divergent evolution on different ages.

Figure 10

Mortality rate at 65 year for Romania



Further, based on the estimated mortality rates we generated prospective life tables and we calculated the life expectancy at birth, life expectancy at 65 and the actuarial present value of a life annuity starting at 65. The summary of the results are shown on tables 1, 2 and 3.

Table 1

Life expectancy at birth calculated based on the forecasted mortality rates fitted with LC method

	Life expectancy at birth					
	Females			Males		
Actual age	RO	BG	HU	RO	BG	HU
30	81,28	79,76	81,26	70,39	68,67	70,42
40	79,2	78,37	79,18	69,09	67,76	68,43

Table 2

Life expectancy at 65 calculated based on the forecasted mortality rates fitted with LC method

	Life expectancy at 65 years					
	Females			Males		
Actual age	RO	BG	HU	RO	BG	HU
30	22,78	18,1	22,45	18,24	15,21	17,02
40	21,74	17,76	21,55	17,49	14,92	16,34

Table 3

Actuarial present value of a life annuity calculated based on the forecasted mortality rates fitted with LC method

	APV life annuity					
	Females			Males		
Actual age	RO	BG	HU	RO	BG	HU
30	17,99	15,34	17,93	14,78	13,01	14,09
40	17,35	15,09	17,35	14,32	12,8	13,65

The interest rate used to calculate the actuarial present value for the life annuities is 2%. Additional risk loadings or fees were not considered.

Actuarial Present Value (or APV) is the expected value or certainty equivalent, of the present value of a conditional cash flow stream (i.e. a series of random payments). Actuarial present values are typically calculated for the series of payments associated with life insurance and life annuities. The probability of a future payment is based on assumptions about the person's future mortality which is typically estimated using a life table. (Bowers, N.L. et all, 1997).

4. Comparison between results of the three models used for forecasting the mortality rates for Romania

We have used the LC method, the Booth-Maindonald-Smith method (BMS) and the coherent functional demographic model of Hyndman, Booth & Yasmeen to forecast the mortality rates by age and cohort. Based on the estimated mortality rates we generated prospective life tables for each method and we calculated the life expectancy at birth, life expectancy at 65 and the actuarial present value of a life annuity starting at age of 65. The interest rate considered was 2% p.a. constant, and no other fees or reserves were considered. The summary of the results are shown on tables 4, 5 and 6.

Figure 10

Life expectancy at birth - by cohort (years)

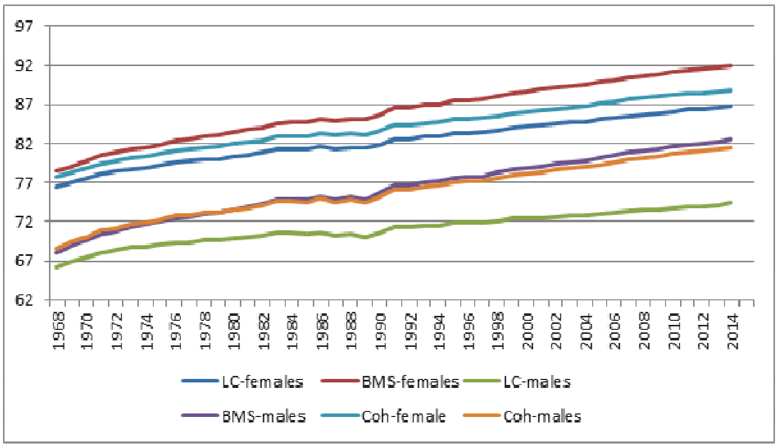


Figure 11

Life expectancy at 65 - by cohort (years)

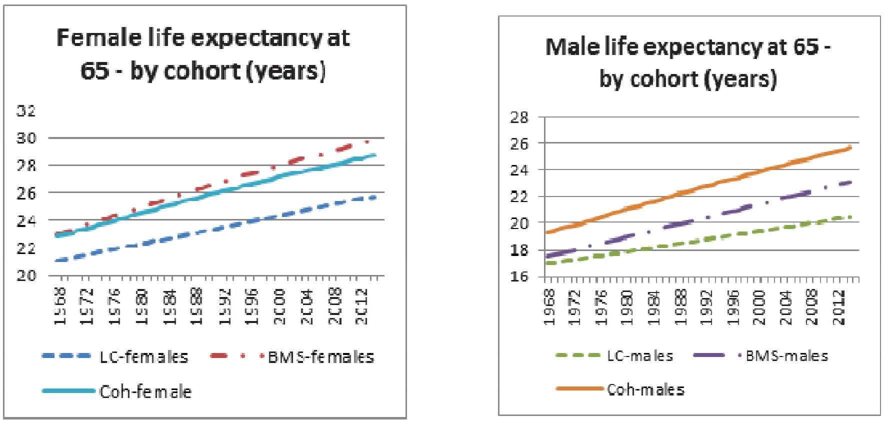


Figure 12

Life expectancy gap between males and females - by cohort (years)

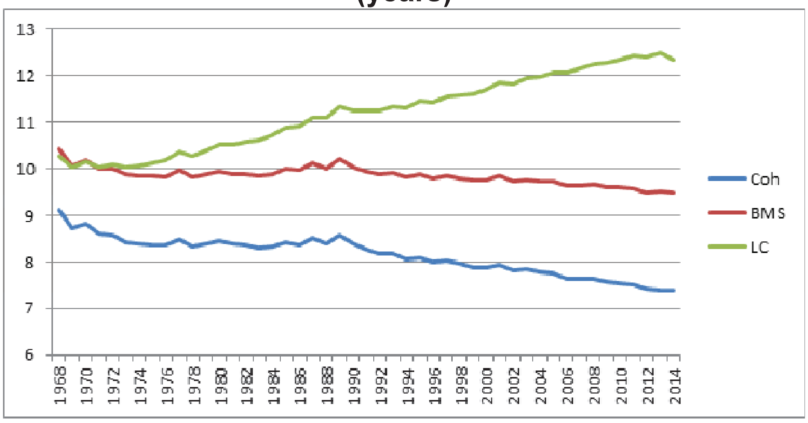


Table 4

Life expectancy at birth calculated based on the forecasted mortality rates fitted with LC, BMS and Coherent functional demographic model

	Life expectancy at birth					
	Females			Males		
Actual age	LC model	BMS model	Coherent model	LC model	BMS model	Coherent model
30	81,28	84,77	82,95	70,39	74,77	74,53
40	79,20	81,88	80,65	69,09	72,04	72,29
47	76,43	78,44	77,71	66,14	68,02	68,60

Table 5

Life expectancy at 65 calculated based on the forecasted mortality rates fitted with LC, BMS and Coherent functional demographic model

	Life expectancy at 65 years					
	Females			Males		
Actual age	LC model	BMS model	Coherent model	LC model	BMS model	Coherent model
30	22,78	25,71	25,18	18,24	19,56	21,76
40	21,74	24,12	23,79	17,49	18,35	20,29
47	21,01	22,97	22,82	16,98	17,51	19,30

The interest rate used to calculate the actuarial present value for the life annuities is 2%.

Table 6

Actuarial present value of a life annuity calculated based on the forecasted mortality rates fitted with LC, BMS and Coherent functional demographic model

	APV life annuity					
	Females			Males		
Actual age	LC model	BMS model	Coherent model	LC model	BMS model	Coherent model
30	17,99	19,75	19,04	14,78	15,70	16,78
40	17,35	18,79	18,21	14,32	14,93	15,88
47	16,89	18,09	17,64	13,99	14,40	15,27

The interest rate used to calculate the actuarial present value for the life annuities is 2%. Additional risk loadings or fees were not considered.

Conclusions

For all three models used in this paper the trend for mortality rates in Romania is decreasing on the following decades. However, there are some differences on how fast the mortality rates will decrease.

Considering the fact that additional risk loadings or fees were not considered, it is possible that the actuarial present values (APV) of the annuities mentioned in this paper to be higher than the calculated values with up to 10%.

In case of generation that has actual age 30 years, in order to get a life annuity at the age of 65 they will need 19.75 times the annual amount they want to receive until death, in case of females, using the BMS model. In case of males, they will need 16.78 times the annual amount they want to receive until death Coherent model.

We could assume that the annuity provider will try to use the most conservative model in order to be on the safe side otherwise they will have to commit capital and the shareholders of the annuity provider will not be pleased.

Referring to generation with actual age 47 years, were the accumulated amount needed is 18.09 times for females and 14.40 times for males, this is due to the difference in life expectancy at 65 age according with BMS model. Furthermore, if risk buffers are considered for longevity, interest rates and other risks than the accumulated amount needed will be around to 20 times the annual amount they want to receive until death, for females and around 16 times for males.

Due to the increasing trend in life expectancy the individuals having the age of 30 years will need 9% more in the accumulated amount at 65 years in order to get a life annuity, comparing with the individuals having the age of 47.

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