SYSTEMIC RISK AND COJUMPS IN HIGH FREQUENCY DATA

Radu LUPU, PhD*
Alexandra MATEESCU, PhD Candidate**

Abstract

Univariate jump detection procedures have been widely studied in the field of statistics of high frequency data, whereas the extension of jump detection to a multivariate framework, in order to understand the correlation between asset returns, is more recent. Cojumps refer to the joint occurence of extreme price movements. The identification of cojumps is extremely important for investors who usually own portfolio of assets. Decisions regarding portofolio allocation, risk management, hedging and pricing can be based on this analysis. The objective of this paper is to investigate the existence of cojumps in European financial market, employing data on the shares of 12stock market indexes. The situations with identified cojumps will be used to identify simultaneous reactions of these markets in order to develop a measure of the systemic risk.

Keywords: jumps, cojumps, simultaneity indicator, high frequency data

JEL Classification: C10, C20, C30, C49

1. Introduction

Risk quantification is one of the most important research fields in finance and financial econometrics which received special attention in the academic literature enjoying, therefore, rapid advances during the last decades. Recently, most of the studies are focusing on the exploitation of high frequency data in order to measure the financial assets price volatility whose understanding and estimation have an important role in financial management. In their activity, investors should bear in mind not only the expected return on investment, but

* *Professor, Faculty of Economics and International Affairs, The Bucharest University of Economic Studies, Bucharest, Romania.*
** *School of Advanced Studies of the Romanian Academy, Bucharest, Romania.*

also the exposure of their strategy to risk, especially during periods of high volatility.

During the last decades, more and more studies based on high frequency data have developed tools in order to measure volatility, which have proved to be extremely efficient form the statistical point of view, helping policymakers, traders and regulators. Thus, a new strand of techniques has emerged, capable of disentangling the so-called “jumps”, i.e. sudden and sharp price discontinuities, generally determined by the arrival of new information in the market. These new developments suggest a clear distinction between the continuous component in asset prices which generates a type of risk that can be easily modelled and predicted and the discontinuous or unpredicted one, generated by jumps.

The empirical evidence suggests that investors should price the expected variation in assets returns differently based on their nature, sharp or continuous price movements. Since these two types of risks have distinct implications, they need to be analyzed and managed accordingly.

The identification of jumps in asset prices is essential for various reasons. Aït-Sahalia (2004) has identified some of them. First of all, in the area of derivatives pricing they play an essential role since investors must take into consideration the presence of a discontinuous price component when establishing their hedging strategies. Moreover, they have a significant role in portfolio allocation. Both continuous and discontinuous components of the risk associated to financial assets should be treated with special attention when deciding the appropriate portfolio management technique. The discontinuous component is uncertain and is usually triggered by the information that flows into the market. Sometimes, more assets are responding the news that enter the market. In this case, we can talk about financial instruments that display common jumps, this phenomenon being called co-jumping among researchers. In this case, it is also extremely important to determine how news affects assets prices, what kind of information is relevant, and how markets process that information. Last, jumps involve major changes in asset prices leading to an increase in the distribution tails. The presence of jumps actually means the existence of fatter tails. Therefore, when researchers need normally distributed time series, the best solution is to identify, estimate and separate jumps from the continuous component.

2. Literature Review

The importance of sudden changes of price dynamics was studied for the first time by Merton (1976) in continuous time processes with jumps and their identification has always been considered an important econometric problem requiring sophisticated numerical estimation techniques. The paper written by Bardorff-Nielesen and Shephard (2004) however, opened a new stage in the process of jump identification methodology. They introduced the usage of bi-power variance as nonparametric volatility estimator. Their main contribution is the use of bi-power variance as nonparametric volatility estimator and their research has been acknowledged mostly for the technique used in order to detect daily jumps. This methodology is based on the fact that the difference between realized variance (as a measure of integrated volatility for a trading day) and the bi-power variation (as integrated volatility measure and robust estimator for jumps) is a stable distribution variable and allows the identification of jumps in case of statistical significance. Lee and Mykland (2008) developed another test which is using simple logarithmic returns that are standardized with a robust jumps estimator and the obtained results are compared with an adequate threshold in order to detect jumps. Thus, intraday jumps (calculated at 5minutes frequency) are determined by comparing returns with a local volatility measure. What can be defined as an abnormal high return depends on the prevailing volatility level.

Also the analysis of the link between discontinuous price changes and macroeconomic news has been at the heart of research in finance lately. Empirical research shows that macroeconomic news affects financial markets. Andersen et al. (2003, 2007) confirmed the importance of jump detection and the fact that most major jumps can be associated with some macroeconomic events. Duffie, Pan and Singleton (2000), Liu, Longstaff and Pan (2003), Eraker, Johannes and Polson (2003) and Piazzesi (2005) emphasize the importance of understanding the causes which lead to jumps in the financial management.

Although there is a relatively large number of non-parametric jump tests, only a limited literature has extended the analysis to a multivariate framework, focusing on the detection of cojumps. Cojumps refer to the joint occurence of extreme price movements. The identification of cojumps could benefit the owners of portfolios since assets prices may display similar or different patterns. Progress in this regard has been made by Barndorff-Nielsen and Shephard

(2003), Gobbi and Mancini (2007), Jacod and Todorov (2009), Bollerslev, Law, and Tauchen (2007) who developed tests for identification of cojumps in a pair of asset returns. In addition to these tests, more research has been made in order to divide cojumps into systematic, meaning cojumps involving the market and idiosyncratic, cojums which exclude the market.Gilder, Shackleton, Taylor (2014) demostrate the connection between jumps in the market portfolio and the cojumps in the independent underlying stocks. They prove that market-level news are able to cause significant cojumps in individual assets.The only event that was associated with systematic cojumps was the Federal Funds Traget Rate announcement. Also, Lahaye, Laurent, Neely (2010) search evidence for cojumps in asset prices and relate them to macroeconomic news announcements. Moreover, Dungey and Hvozdyk (2011) examined the behaviour of bonds, both spot and futures markets, in order to determine the existence of common jumps. The results showed that joint jumps occur mostly in the case of instruments with shorter maturities. The authors also determined that the probability of simultaneous jumps is affected largely by news surprises in non-farm payrolls, consumer price index (CPI), gross domestic product (GDP) and retail sales. Another example of cojumps test is proposed by Liao and Anderson (2011) who use a return-based cojump test developed by Bollerslev et al (2007), a range-based cojump test and a first-high-low-last (FHLL) price based cojump test in order to analyze the cojumps in each stock and the cojumps across the two stock exchanges.

3. Data and methodology

The data used for the analysis consists of five-minute stock market index returns from some of the developed European markets such as: Germany (DAX), France (CAC), United Kingdom (UKX), Portugal (PSI20), Spain (IBEX), The Netherlands (AEX), Sweden (OMX), Italy (FTSEMIB), Austria (ATX), Switzerland (SMI), Belgium (BEL20) and Ireland (ISEQ). We took into account a period of approximately 6 months, starting from 21st of April 2016 until the 2nd of November 2016. Price data was obtained through Bloomberg platform, and the analysis was performed in Matlab.

The purpose of this study is to determine the existence of cojumps in the data series of previously mentioned indexes prices. This methodology involves, first, the identification of jumps moments for each individual data series. Then we identify the common jumps (cojumps) among the 12 European indexes and based on these results we build an indicator of simultaneity.

However, in order to offer an accurate jump estimation, it is necessary to eliminate first the periodicity component from the data series. A time series is periodic if it has a regular, time-dependent structure. Volatility in assets prices could display a periodic pattern determined by regular trading trends. Financial assets price volatility usually displays a periodic pattern caused by regular trading trends or effects of regular macroeconomic news as pointed out by Erdemlioglu, Laurent and Neely (2012).

Because of these regular variations, the variance of returns computed for high frequency data, $\sigma_{t,i}^2$, has a periodic component, $f_{t,i}^2$. Erdemlioglu, Laurent and Neely (2012) assume that $\sigma_{t,i}^2 = s_{t,i} f_{t,i}$, where $s_{t,i}$ is the stochastic part of the intraday volatility which is constant within one day, but varies form one day to another and $f_{t,i}$ is the standard deviation periodicity.They propose the following estimator of the standard deviation periodicity: $\hat{f}_{t,i}^{SD} = \frac{SD_{t,i}}{\sqrt{\frac{1}{M}\sum_{j=1}^{M} SD_{t,i}^2}}$, where $SD_{t,i} = \sqrt{\frac{1}{n_{t,i}}\sum_{j=1}^{n_{t,i}} \bar{y}_{j;t,i}^2}$. Therefore the log-returns used in this analisys are periodicity-adjusted returns, i.e. returns divided by the $f_{t,i}$ measure of periodicity.

After the data is adjusted and the periodic component is removed, the next step in our analysis consists in the identification of jumps.

In order to determine whether a return is very high (it has an "abnormal" value), we need to analyze the prevailing level of volatility in a given period of time. Thus, in periods of high volatility, an "abnormal" return is higher than an "abnormal" return in periods with low volatility.

The technique used for jump identification is based on the methodology proposed by Lee and Hannig (2010). The test applied in this this section is used in order to determine jumps at a certain moment t_j, where *t*is the day and *j* is the 5-minute interval within that day. The test is built starting from the null hypothesis which assumes that there are no jumps at a given moment in time t_j. This allows the identification of the exact time of jump occurrence. This procedure is called "intraday" test as it can detect jumps at any time during a trading day, and not only at a daily level.

The specification of the test is the following:

$$J_{t,i}^{LH} = \frac{|y_{t,i}|}{\hat{\sigma}_{t,i}}$$

where $J_{t,i}^{LH}$ is the jump test, t is the time-frame used for the computation of our analysis, i.e. the time sample (usually it has the size of a day), while *I* counts the moments in this time-frame. The $\hat{\sigma}_{t,i}$ is the standard deviation computed for this time sample and is actually replaced by the estimated standard deviation$\hat{s}_t = \sqrt{\frac{1}{M-1}TV_t}$, according to the methodology used by Lee and Hannig.$TV_t$is the Truncated Variation and is given by the following equation:

$$TV_t(\Delta) \equiv \sum_{i=1}^{M} \left(y_{t,i}\right)^2 1_{|y_{t,i}| \leq g(\Delta)^{\varpi}} \rightarrow \int_{t-1}^{t} \sigma^2(s)ds$$

where g > 0and ϖ ∈ (0, ½) are used for the computation of the thresholds needed to eliminate the large returns from the series used in the computation of the volatility and use only those that are lower that the specified threshold. For the estimation of TV_t we use the following values g = 0,3·9 and ϖ = 0,47, according to the methodology proposed by Aït-Sahalia and Jacod (2009b).

For a more accurate estimation of the prevailing volatility, we brought an improvement to this methodology. We eliminated also the returns with 0 value. Thus, the resulting prevailing volatility does not take into account neither the returns with an extremely high value which exceeds the threshold previously imposed, nor the very low returns that could cause an erroneous estimation of the prevailing volatility within a certain time frame.

Also, to minimize the risk of detecting false jumps, the authors try to establish how big the statistic can become in the presence of jumps. If the statistic exceeds a plausible maximum, the null hypothesis of no jump is rejected. Under this framework and the absence of jumps within [*i-1, i*] from day *t*, then when Δ→ 0the sample maximum of the absolute value of a standard normal variable (that is the jump statistic $J_{t,i}^{LH}$) follows a Gumbel distribution. Therefore, the null hypothesis (no jump) is rejected if:

$$J_{t,i} > G^{-1}(1-\alpha)S_n + C_n$$

where $G^{-1}(1-\alpha)$is the quantile function$1-\alpha$ of the standard Gumbel distribution, and

$$C_n = (2\log n)^{0.5} - \frac{\log(\pi) + \log(log\ n)}{2(2\log n)^{0.5}}$$

$$S_n = \frac{1}{(2\log n)^2}$$

After we compute the jumps for the data series of each company, we identify the common jumps.

The analyzed period is divided in 26 weeks. Every week, within the same time frame at five-minute frequency, we computed the number of cojumps for the 12 European indexes. We remove from the analysis the moments when no cojumps were detected, i.e. at a given moment we found zero or only one jump among the share returns of the 12 indexes. We consider only the moments when we detected at least two simultaneous jumps.

For each week ($week_i$) we compute the simultaneity indicator which takes the following form:

$$Simultaneity\ indicator_{week_i} = \frac{\sum_{j=1}^{12} co - jumps \times \frac{n_{co-jumps}}{N}}{12}$$

where $co - jumps$ is the number of common jumps which can be obtained in a certain time frame. This takes values from 1 to 12, where 12 is the total number of companies in our sample. $n_{co-jumps}$ corresponds to the number of situations in which we had a number of simultaneous jumps equal to the value of $co - jumps$ and N is the total number of situations in which we acknowledged at least two jumps happening simultaneously in one week.

Computed in this way, the indicator can take values between 0 and 1, being a measure of simultaneity within the share prices of the 12 indexes in our sample and also a measure of systemic risk in the European financial market. For example, if we determine only individual jumps, i.e. none of the jumps identified in a specific time frame was simultaneous among indexes, then the indicator would have the values 1/12. It will be equal to zero when no jump is detected and equal to one when we identify cojumps among all the 12 data series, at the same moment. Therefore, the value of 1 represents perfect simultaneity while the value of 1/12 perfect independence.

Moreover, this indicator is computed both for the situations when we identify negative jumps and for the situations when we identify positive jumps.

4. Results

The results which were obtained from the previously presented models are exhibited in Figures 1 and 2. These figures show the evolution of simultaneity indicator for each week of the analyzed period. In the first figure we present the indicator computed for negative cojumps, while the second one shows the same indicator computed for positive cojumps. It can be observed that in both cases the simultaneity indicator tends to be in the same range of values for the entire period. Moreover, both indicators are displaying similar values, which demonstrates the existence of simultaneity.

Figure 1

The evolution simultaneity indicator for negative cojumps across the 26 weeks in our sample

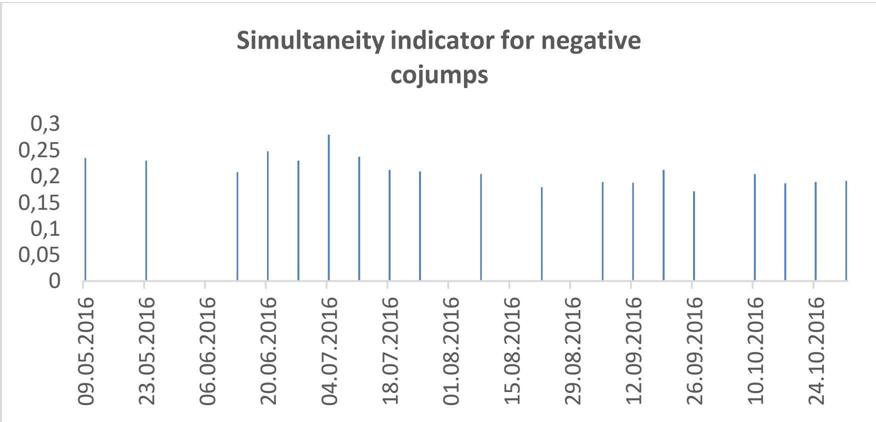
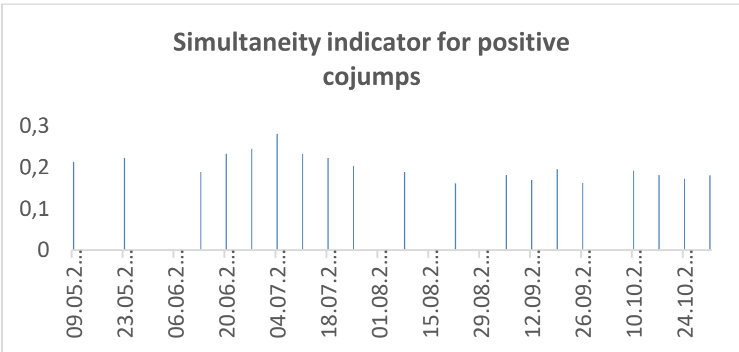


Figure 2

The evolution simultaneity indicator for positive cojumps across the 26 weeks in our sample



The simultaneity indicator displays higher values for both positive and negative jumps during weeks when major economic events took place. For instance, the speech given by European Central Bank's president generated substantial movement across the European financial market in April and June. Macroeconomic events such as publication of inflation rate, economic sentiment index or consumer confidence also caused common jumps among the analyzed index prices.

Moreover, previous theoretical and empirical literature on asset returns demonstrate that usually markets respond more to negative news in good times. Even though our analysis is based on jumps rather that returns, it can be observed from the presented results that negative jumps are more persistent. These results are consistent with the ones confirmed by Lupu (2014) in a previous study that approaches this topic.

5. Conclusions

The decision making process in finance is very complex because of the high level of uncertainty in any financial market. Therefore, the accuracy of the models used to estimate the volatility has increased recently. The analysis presented in this paper highlights the great importance of using high frequency data in order to estimate volatility and correlations among financial assets. Jumps have an important role in the quantification of financial risk because they allow the separation and the differential analysis of the continuous and discontinuous price components. Moreover, the phenomenon of market co-dynamics has gained a lot of attention lately, cojumps being important indicators of systemic risk in a financial market.

This paper contributes to identification of co-dependence for a sample of 12 index returns and it proposes a new methodology for the estimation of simultaneity in the stock market. This type of analysis can be extremely useful to investors in the management of local portfolios and risk management.

The results computed both for positive and negative jumps display a relatively high level of simultaneity among the shares of the 12 European indexes in our sample which indicates the fact that prices adjust rapidly and respond simultaneously to any new information that enters the market.

References

1. Aït-Sahalia Y., and Jacod J. (2009b), “Testing for Jumps in a Discretely Observed Process”, *Annals of Statistics*, vol. 37 (1), pp. 184–222.
2. Andersen T. G., Bollerslev T., Diebold F. X., and Vega, C. (2003), “Micro effects of macro announcements: Real-time

price discovery in foreign exchange", *The American Economic Review* vol. 93, pp. 38-62.

3. Andersen T., T. Bollerslev and D. Dobrev (2007), "No-arbitrage Semi-martingale Restrictions for Continuous-time Volatility Models Subject to Leverage Effects, Jumps and i.i.d. noise: Theory and Testable Distributional Implications", *Journal of Econometrics*, vol. 138, pp. 125–180.
4. Aït-Sahalia Y. (2004),"Disentangling diffusion from jumps", *Journal of Financial Economics* vol. 74, pp. 487–528.
5. Barndorff-Nielsen O. and Shephard N. (2004), "Power and bipower variation with stochastic volatility and jumps", *Journal of Financial Econometrics*, pp. 1–48.
6. Barndorff-Nielsen O. E. and Shephard N. (2003), "Econometrics of testing for jumps in financial economics using bipower variation", *Unpublished discussion paper: Nuffield College, Oxford.*
7. Bollerslev T., Law T. H. and Tauchen G. (2007),"Risk, jumps, and diversification", *Working paper, Duke University*.
8. Duffie D., Pan J. and Singleton K. (2000), "Transform Analysis and Asset Pricing for Affine Jump Diffusions", *Econometrica*, vol. 68, pp. 1343-1376.
9. Dungey M. and Hvozdyk L. (2011), "Cojumping: Evidence from the US Treasury bond and futures markets", *Journal of Banking & Finance*, vol. 36, pp 1563-1575.
10. Eraker B., Johannes M. and Polson N. (2003), "The Impact of Jumps in Volatility and Returns", *Journal of Finance*, vol. 53, pp. 1269-1300.
11. Erdemlioglu, D., Laurent S. and Neely C. J. (2012), "Econometric modeling of exchange rate volatility and jumps", Working Paper 2012-008A, Federal Reserve Bank of St. Louis.
12. Gilder D., Shackleton M., Taylor S. J. (2014), "Cojumps in stock prices: empirical evidence", *Journal of Banking and Finance*, vol. 40, pp. 443-459.
13. Gobbi F. and Mancini C. (2007),"Identifying the covariation between the diffusion parts and the co-jumps given discrete observations", *Dipartimento di Matematica per le Decisioni*, Universita degli Studi di Firenze.
14. Jacob J. and Todorov V. (2009), "Testing for common arrivals of jumps for discretely observed multidimensional processes," *Annals of Statistics*, vol. 37, pp. 1792-1838.

15. Lahaye S., Laurent L. and Neely C. (2010), “Jumps, cojumps and macro announcement”, *Journal of Applied Econometrics*, vol. 26, pp. 893–921.
16. Lee S. S. and Mykland P. A. (2008), “Jumps in Financial Markets: A New Nonparametric Test and Jump Dynamics”, *Review of Financial Studies*, vol. 21, pp. 2535–2563.
17. Lee, S. S., and J. Hannig (2010), “Detecting Jumps from Lévy Jump-Diffusion Processes”, *Journal of Financial Economics*, pp. 271–290.
18. Liao Y. and Anderson H. M. (2011) "Testing for co-jumps in high-frequency financial data: an approach based on first-high-low-last prices", *Monash Econometrics and Business Statistics Working Papers 9/11*, Monash University, Department of Econometrics and Business Statistics.
19. Liu J., Longstaff F. and Pan J. (2003), “Dynamic Asset Allocation with Event Risk”, *Journal of Finance*, vol. 58, pp. 231–259.
20. Lupu R. (2014), “Simultaneity of Tail Events for Dynamic Conditional Distributions”, *Romanian Journal of Economic Forecasting*, Volume 17, pp. 49-64
21. Merton R. (1976), “Option pricing when underlying stock returns are discontinuous”, *Journal of Financial Economics*, pp. 125–144.
22. Piazzesi M. (2005), “Bond yields and the federal reserve”, *Journal of Political Economy*, vol. 113, pp. 311– 344.