DEVELOPMENT OF PERIODIC LOAN REPAYMENT MODELS CONSIDERING RHYTHMIC SKIPS

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Abstract

The notion of loan repayments rest on the principle that present value of sum of the instalments are equal to present value of the loan total. In a standard loan repayment plan, periodic instalments are set to a fixed amount. Besides, loan repayment plans with geometrically or arithmetically increasing periodic amounts can also be found at mathematics of finance textbooks. Beyond that models for loan repayments with skips deal with types of loans in that some periods are predetermined to pass by without making any instalment. Payment skips in some periods have been requested by some clients as expenses in some months rise considerably. In this study, general formulas are derived under the assumptions that periodic loan repayments adjust with arithmetic gradient series and interrupt with rhythmic non-payment periods. Later, by setting the arithmetic change to zero, a general formula for loan repayment with equal periodic instalments that also has rhythmic skips has been derived. Same numerical examples with solutions are presented for the developed models. As a result numerical examples have been used in order to show the validity of the models.

Keywords: Loan Amortization, Periodic Payments, Skip Periods, Rhythmic Skips

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1. Introduction

Engineering economics plays a significant role in decision sciences (Blank and Tarquin, 2005). The cash flows, time value of

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money and interest rates are the most substantial research area in mathematical finance (Aydemir and Eroğlu, 2014: 95; Parvez, 2005). It is conventional that bank loans are repaid in equal periodical instalment amounts. It is essential that financial institutions offer alternative loan repayment plans that fit individuals income flow, which may decrease and increase along the year depending on the months or seasons. The notion of loan amortization rests on the principle that present value of a loan is equal to sum of periodic payments. The problems with payments of a debt stand with equality of the debt's present value and the sum of the present values (Shao and Shoa, 1998). In addition, arithmetic and geometric change models are available in financial mathematics textbooks. In mathematical finance textbooks general formulas are available for the cases where periodic payments are kept equal or are set to change in geometric or arithmetic sequences along the term (Park, 1997: 55).

The loan repayment model with equal periodic payment is:

$$d = \frac{pr}{1 - R^{-n}} \tag{1}$$

The loan repayment model where the periodic payments change in geometric sequence along the term is:

$$d_k = dG^{k-1} \quad k = 1, 2, n \tag{2}$$

$$d = \begin{cases} \dfrac{p(r-g)}{1-i^n} & , \quad g \neq r \\ \dfrac{pR}{n} & , \quad g = r \end{cases} \tag{3}$$

The loan repayment model where the periodic payments change in arithmetic sequence along the term is (Eroglu, 2000):

$$d_k = d + (k-1)v \quad , \quad k = 1, \cdots, n \tag{4}$$

$$d = \frac{pr^2R^n + v\left[1 + nr - R^n\right]}{r\left(R^n - 1\right)} \tag{5}$$

Where:
d: instalment or periodic payment amount,
d_k: k^{th} period due payment amount,
n: number of periods,
p: loan amount,
r: periodic interest rate, $R = 1 + r$
g: proportionate change (geometric) in periodic payment amount and $G = 1 + g$, $i = GR^{-1}$
v: absolute change (arithmetic) in periodic payment amount.

After a certain time from it takes credit, the customer's ability to pay may change. Therefore the customer may want to determine own the amount of a certain number of instalments in the first few months. Above three models assume that periodic payments are made at the end of the period. Formato (1992) developed a model in which client asks for not to pay make payment in certain periods that he/she would choose, for instance due to vacation expenses. Formato's skip payment model extended to the case where periodic payments change in geometric sequence by Moon (1994) and to extend to the case where periodic payments change in geometric sequence by Eroglu and Karaoz (2002). General formulae for cases when outstanding instalments have regular or irregular parts and geometric or arithmetic changes from one period to another were first discussed by Eroglu (2000). Moreover, Eroglu (2001) developed skipped payment models where periodic payments changes in partially geometric and arithmetic sequences and skipped payments are arbitrary. In fact, all four studies the skip periods, in which instalments are not made, are chosen arbitrarily. Arbitrary skips in instalments impose that periodic payments halt with occasional non-payment periods which may also differ in duration (Eroglu and Ozturk, 2016). Here, at which cycles the instalments will be skipped in the models in question is selected randomly. In addition to the randomly skipped loan payment models, rhythmically skipped loan payment models were discussed by Eroglu et al. (2011), Eroglu and Ozdemir (2012) and Eroglu et al. (2013). On the other hand, a loan payment model of which the certain number of instalments in the first months is determined by the customer was explained with another approach by Eroğlu (2013a). This model was further developed by Eroglu (2013b), Eroglu et al. (2014) and Eroglu and Ozturk (2016) was addressed as a loan model in which the certain number of instalments is identify by the customer at the beginning and then the number of instalments indicates arithmetic changes, and as a loan payment model in which the number of instalments indicates the partial geometric change for periods created by the equal instalment cycles.

The purpose of this study is to provide an alternative repayment model for the debts owed to a credit institution or to a bank through developing a novel mathematical model in line with the demands of the consumers.

In this study, general formulas will be derived under two assumptions about periodic payments. First periodic payments adjust with arithmetic sequence. Second periodic payments halt with rhythmic skip intervals along the loan term. Later, by setting the arithmetic change to zero, a general formula for loan repayment with equal periodic instalments that also has rhythmic skips will be derived. Numerical examples will be used in order to show the practicalities of the models.

2. A model for loan repayment with periodic payments that has arithmetic change and rhythmic skips

Fixed payment models are commonly used model for the loan amortization payments that are given from credit institutions or banks. In current life, after a certain period from the repayments was started, the customer's payment facility could be involved from the acquisition of the variability. In this case, the customers want to change and make easy the instalment level in the amount of a certain number of beginning periods itself for considering under changes in income for the coming periods (Aydemir and Eroglu, 2014: 97).

In this model, loan amortization is made with periodic payments. We define two intervals, namely instalment interval and skip interval. Instalment interval is the time interval which only contains periods with successive periodic instalments without interruption or skipped. Similarly, skip interval is the time interval that only contains successive periods in which periodic payments are stopped. We assume that a loan repayment schedule or loan term has at least two instalment intervals. Thus a loan term starts and ends with instalment intervals, which all have same length or contains same number of payment periods. Skip intervals are located between the instalment intervals and are equal to each other as well in number of periods without pay. Thus, we assume that periodic payments halt with orderly non-payment periods which are also identical in duration, which makes the skips rhythmic rather than arbitrary. In contrary, arbitrary skips in instalments assumption impose that periodic payments halt with occasional non-payment periods which may also differ in duration. Rhythmic skip assumptions makes our proposed models differ from prior studies.

Following additional notations are used in our model.

f: Total number of payment periods in an instalment interval.

h: Total number of periods in a skip interval.

M_k: First period number of an instalment interval that comes right after k^{th} skip interval.

L_{k+1}: Last period number of an instalment interval that comes right after k^{th} skip interval.

d_{kj}: The total amount of periodic payments made at j^{th} instalment interval that comes right after a skip interval.

s: Total number of skip intervals.

n: Duration of loan repayment schedule in number of periods.

We assume each instalment interval equal to each other. Same rule applies to skip intervals as well. Then, following expressions can be written:

$$M_k = k(f+h)+1, \qquad k=0,....,s$$
$$L_{k+1} = k(f+h)+f \qquad k=0,....,s$$
$$n = L_{s+1} = s(f+h)+f$$

On the other hand instalment amounts follow an arithmetic adjustment process:

$$d_{kj} = d + (j + k - 1 + Y_k)v = d + (j - 1 - kh)v \qquad \begin{matrix} k=0,\ldots,s \\ j=M_k,\ldots,L_{k+1} \end{matrix} \qquad (6)$$

Where:

$$Y_k = \sum_{t=1}^{k} L_t - M_t = \sum_{t=1}^{k}\left[(t-1)(f+h)+f-t(f+h)-1\right] = -k(h+1),$$

Formula can be derived. Since present value of the loan are equal to present value of sum of all instalments,

$$\rho = \sum_{k=0}^{s} \sum_{j=M_k}^{L_{k+1}} d_{kj} R^{-j} = dA + vB \qquad (7)$$

is achieved.

Where:

$U_1 = \left(R^{-f} - 1\right)$, $U_2 = \left(R^{-(f+h)} - 1\right)$, $U_3 = \left(R^{-(f+h)(s+1)} - 1\right)$, $A = \frac{-U_1 U_3}{rU_2}$,

$B = \frac{fU_1}{rU_2}\left[U_3\left(\frac{1}{U_2} - \frac{1}{fr} - 1 - s - \frac{1}{U_1}\right) - 1 - s\right]$ (see Appendices)

Considering equation (7), following formulas are achieved:

$$d = \frac{p - vB}{A} \qquad 8)$$

$$v = \frac{p - dA}{B} \qquad 9)$$

Equations (8) and (9) are general formulas or models derived under rhythmic skip assumption.

Numerical Example 1

An automobile has a price tag of 60000 dollars. Yet it has been purchased with monthly repayment loan. A loan schedule will be set up with 10 months periodic payments and then 2 months of skips for a total of 34 months. Periodic payment amounts will increase with 10 dollars from one instalment to another successively, excluding skip periods. We calculate the monthly instalment amounts setting up the monthly interest rate to %1.

Then problem inputs can be summarized as below:

p: 60000, f: 10, h: 2, s: 2, n: 34, v: 10, r: 0.01

Using equation (8), the first instalment amount is calculated as d = 2231.945. Later, other instalment amounts are calculated from equation (6).

Table 1

Loan Repayment Schedule for Numerical Example 1

Months	Instalment (periodic payment) amounts	Balance due (dollars)
0	-	60000
1	2231.945	58368.055
2	2241.945	56709.791
3	2251.945	55024.943
4	2261.945	53313.248
5	2271.945	51574.435
6	2281.945	49808.235
7	2291.945	48014.372
8	2301.945	46192.571
9	2311.945	44342.552
10	2321.945	42464.032
11	0	42888.672
12	0	43317.559
13	2331.945	41418.790
14	2341.945	39491.033
15	2351.945	37533.998
16	2361.945	35547.393
17	2371.945	33530.922
18	2381.945	31484.286
19	2391.945	29407.184
20	2401.945	27299.311
21	2411.945	25160.359
22	2421.945	22990.017
23	0	23219.918
24	0	23452.117
25	2431.945	21254.693
26	2441.945	19025.295
27	2451.945	16763.603
28	2461.945	14469.294
29	2471.945	12142.042
30	2481.945	9781.517
31	2491.945	7387.387
32	2501.945	4959.316
33	2511.945	2496.964
34	2521.945	0

3. A Model for Loan Repayment with Equal Periodic Payments and Rhythmic Skips

Taking v = 0, the model for loan repayment with periodic instalments that has arithmetic change and rhythmic skips will reduce to a model for loan repayment with equal periodic payments and rhythmic skips. Then general formulas of (7) and (8) will modify to (10) and (11):

$$p = \frac{d\left(1 - R^{-f}\right)\left(R^{-(f+h)(s+1)} - 1\right)}{r\left(R^{-(f+h)} - 1\right)} \qquad 10)$$

$$d = \frac{rp\left(R^{-(f+h)} - 1\right)}{\left(1 - R^{-f}\right)\left(R^{-(f+h)(s+1)} - 1\right)} \qquad 11)$$

Numerical Example 2

An automobile which has the advance price of 45000 dollars has been purchased with a loan schedule which includes consecutive 5 months periodic payments and a month of skip for a total of 23 months. We calculate the monthly periodic payment amounts setting up the monthly interest rate to %1.

The problem inputs are given in below:

p: 45000, f: 5, h: 1, s: 3, n: 23, r: 0.01

Using the equation (11), the monthly instalment amounts become d = 2529.467.

Table 2

Loan Repayment Schedule for Numerical Example 2

Months	Instalment (periodic payment) amounts	Balance due (dollars)
0	-	45000
1	2529.467	42920.533
2	2529.467	40820.271
3	2529.467	38699.007
4	2529.467	36556.530
5	2529.467	34392.628
6	0	34736.555

Months	Instalment (periodic payment) amounts	Balance due (dollars)
7	2529.467	32554.453
8	2529.467	30350.531
9	2529.467	28124.569
10	2529.467	25876.348
11	2529.467	23605.644
12	0	23841.701
13	2529.467	21550.651
14	2529.467	19236.690
15	2529.467	16899.590
16	2529.467	14539.119
17	2529.467	12155.043
18	0	12276.594
19	2529.467	9869.893
20	2529.467	7439.125
21	2529.467	4984.049
22	2529.467	2504.422
23	2529.467	0

4. Conclusion

Current economies are lay out with higher variability in the cases from globalism. So, the customer types and the debt volumes are also changed in globally markets. The customers or investors are demanded the new repayment financial models for their variable debts levels market conditions. As a result of these changes and innovations, the new repayment models must be derived in alternatively for loan payment model that most commonly used by banks or credit institutions.

The notion of loan repayments rest on the principle that present value of sum of the periodic payments are equal to present value of the loan total. Loan amortization models differ from each other in distribution of instalments. Equal, geometric gradient and arithmetic gradient periodic payments exist in conventional models. From a customer oriented financial institution's view point, it can be functional and can help creating satisfying customers to offer alternative loan repayment plans that fit individual's personal income flow, which may change along the year. Alternative loan repayment models with alternative periodic payment plans should facilitate reaching to additional customers, to those otherwise do not shop at the market. This idea led to derivation of the loan amortization models that have arbitrary skips by Formato (1992), Moon (1994), Eroğlu (2000), Eroglu and Karaoz (2002), Eroglu et al (2011), Eroglu and Ozdemir (2012), Eroglu et al. (2013), Eroğlu (2013a), Eroglu (2013b) and Eroglu et al. (2014), Aydemir and Eroglu (2014).

In this study, general formulas derived under two assumptions in this current paper. First, periodic payments adjust with arithmetic sequence. Second, periodic payments take breaks with rhythmic skip intervals along the loan term. Later, by setting the arithmetic change to zero, a general formula for loan repayment with equal periodic instalments that also has rhythmic skips has been derived. The results are obtained with the numerical examples with a repayment schedule is shown in clearly and understandable. The results of this study indicate that numerical examples have been used in order to show the validity of the models.

This study is important in terms of the determination of the amount of payment for customers and pays the appropriate balance. This study is also significant in order to increase repayment options in any debt payment model and to access more consumers when these models are applied by banks or credit institutions.

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$$p = \sum_{k=0}^{s} \sum_{j=M_k}^{L_{k+1}} d_{kj} R^{-j} = \sum_{k=0}^{s} \sum_{j=k(f+h)+1}^{k(f+h)+f} [d + (j - 1 - kh)v] R^{-j}$$

$$= \left[\sum_{k=0}^{s} \sum_{j=k(f+h)+1}^{k(f+h)+f} dR^{-j} \right] + \left[\sum_{k=0}^{s} \sum_{j=k(f+h)+1}^{k(f+h)+f} (j - 1 - kh) v R^{-j} \right]$$

$$= d \left[\sum_{k=0}^{s} \sum_{j=k(f+h)+1}^{k(f+h)+f} R^{-j} \right]$$

$$+ v \left\{ \left[\sum_{k=0}^{s} \sum_{j=k(f+h)+1}^{k(f+h)+f} -(1 + kh) R^{-j} \right] + \left[\sum_{k=0}^{s} \sum_{j=k(f+h)+1}^{k(f+h)+f} j R^{-j} \right] \right\} = dA + v(B_1 + B_2)$$

$$= dA + vB$$

Where:

$$U_1 = (R^{-f} - 1)$$

$$U_2 = (R^{-(f+h)} - 1)$$

$$U_3 = (R^{-(f+h)(s+1)} - 1)$$

$$B_1 = \frac{U_1}{rU_2} \left\{ U_3 + h \left[s(U_3 + 1) - \frac{U_3}{U_2} + 1 \right] \right\}$$

$$B_2 = \frac{-(f + h)U_1}{rU_2} \left[s(U_3 + 1) - \frac{U_3}{U_2} + 1 \right] + \frac{U_1 U_3}{rU_2} \left[-1 - \frac{1}{r} - f \left(1 + \frac{1}{U_1} \right) \right]$$

$$A = \frac{-U_1 U_3}{rU_2}$$

$$B = B_1 + B_2 = \frac{fU_1}{rU_2} \left[U_3 \left(\frac{1}{U_2} - \frac{1}{fr} - 1 - s - \frac{1}{U_1} \right) - 1 - s \right]$$

$$B_1 = \sum_{k=0}^{s} \sum_{j=k(f+h)+1}^{k(f+h)+f} -(1+kh)R^{-j} = -\sum_{k=0}^{s} \sum_{j=k(f+h)+1}^{k(f+h)+f} (1+kh)R^{-k(f+h)-1}\left(\frac{R^{-f}-1}{R^{-f}-1}\right)$$

$$= \frac{\left(R^{-f}-1\right)}{r}\left(\sum_{k=0}^{s}(1+kh)R^{-k(f+h)}\right)$$

$$B_1 = \frac{\left(R^{-f}-1\right)}{r}\left[\left(\sum_{k=0}^{s} R^{-k(f+h)}\right) + \left(h\sum_{k=0}^{s} kR^{-k(f+h)}\right)\right]$$

$$= \frac{\left(R^{-f}-1\right)}{r}\left[\left(\sum_{k=0}^{s} R^{-k(f+h)}\right) + \left(h\sum_{k=1}^{s}\sum_{t=k}^{s} R^{-t(f+h)}\right)\right]$$

$$= \frac{\left(R^{-f}-1\right)}{r}\left[\left(\frac{R^{-(f+h)(s+1)}-1}{R^{-(f+h)}-1}\right)\right.$$

$$\left. + \left(\frac{h}{R^{-(f+h)}-1}\left[sR^{-(f+h)(s+1)} - \left(\frac{R^{-(f+h)(s+1)}-R^{-(f+h)}}{R^{-(f+h)}-1}\right)\right]\right)\right]$$

$$= \frac{R^{-f}-1}{r(R^{-(f+h)}-1)}\left\{R^{-(f+h)(s+1)}-1\right.$$

$$\left. + h\left[sR^{-(f+h(s+1))} - \left(\frac{R^{-(f+h)(s+1)}-R^{-(f+h)}}{R^{-(f+h)}-1}\right)\right]\right\}$$

$$= \frac{U_1}{rU_2}\left\{U_3 + h\left[s(U_3+1) - \frac{(U_3+1)-(U_2+1)}{U_2}\right]\right\}$$

$$= \frac{U_1}{rU_2}\left\{U_3 + h\left[s(U_3+1) - \frac{U_3}{U_2} + 1\right]\right\}$$