USING THE SYMMETRIC MODELS GARCH (1.1) AND GARCH-M (1.1) TO INVESTIGATE VOLATILITY AND PERSISTENCE FOR THE EUROPEAN AND US FINANCIAL MARKETS

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Abstract

In this paper, we used the GARCH (1,1) and GARCH-M (1,1) models to investigate volatility and persistence at daily frequency for European and US financial markets. In the study we included fourteen stock indices (twelve Europeans and two Americans), during March 2013 - January 2017. The results of the GARCH (1.1) show that the models are correctly specified for most of the analysed series (except for the WIG30 index). The study found that the BET-BK index recorded the lower persistence of volatility, meaning that the conditional volatility tends to revert faster to the long-term mean than the other stock indices analysed. In the case of the GARCH-M (1.1) model, the variance coefficient in the mean equation was statistically significant and positive (thus confirming the hypothesis that an increase in volatility leads an increase in future returns), only for six of the analysed series. The strongest relationship was recorded for the US index, S&P500. It is also recorded for the Romanian stock indices: BET and BET-BK. For the BET index, the conclusions are in line with the results of previous studies.

Keywords: stock market, volatility clustering, volatility persistence

JEL Classification: C22, C32, C51, G11, G17

1. Introduction

The global financial crisis has made financial markets characterized by a high degree of uncertainty and high volatility in the prices of financial assets, irrespective of their type: stocks, bonds, commodities, derivatives, etc. Volatility makes difficult the anticipation

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of the future evolution of earnings from financial placements and requires increased attention from investors, speculators, fund managers and, last but not least, financial market regulators.

If the return, the risk, the time horizon, and the liquidity of financial placements are notions with which stock market investors are largely familiar, volatility is a more difficult variable to quantify, as it cannot be directly observed. Financial market participants perceive volatility differently, depending on the daily variation in trading prices (decrease or increase). Volatility has been shown to increase as the market recorded significant declines in financial asset prices and is lower when the market is on the rise. As a result of this, the volatility is usually associated by investors with the loss rather than profit, and in this case, they approach with caution the periods of increased volatility. Instead, speculators step up their trading activity during these times, attracted by increased profit opportunities. The volatility behaviour can be analysed through the variation in the return on financial assets. Studies on financial time series have highlighted some of their features such as leptokurtotic distribution, leverage effect, heteroscedasticity, fat tails, volatility clustering, autocorrelation or serial correlation in residuals, etc. The phenomenon of "volatility clustering" visible effect of heteroskedasticity was first observed by Mandelbrot (1963). He concluded that high return variations are followed by major future changes, while low return variations are most likely followed by small fluctuations. Volatility clusters can be observed by analysing the volatility chart of stock indices included in this study. We test the GARCH (1.1) and GARCH-M (1.1) models and analysed fourteen stock indices: twelve Europeans indices (less founded in the specialty studies, including BET and BET-BK) and two Americans indices.

The study period is a more recent one, March 2013-January 2018, and it should be characterized by a lower volatility than the one recorded during the financial crisis.

The selection of the two GARCH (1.1) and GARCH-M (1.1) models is motivated by the conclusions of previous studies on this theme. Hansen and Lunde (2005) showed that a GARCH (1.1) model using only three parameters in the conditional variance equation is sufficient to model the financial series. The study of the applicability of the GARCH-M (1.1) model on this set of stock indices was based than on observation of a positive relation between the assumed risk and the obtained return, relation which can be surprised by a variable introduced in the mean equation of the model. This variable must be positive and statistically significant. A previous study, conducted over the period 1997-2012, for the BET index (which is found in this study) reported the absence of this relationship regardless of the frequency of the data analysed. In addition, the model failed to remove the ARCH effects left of the daily residuals series.

The originality of the study is given both by the analysed period of time (March 2013 - January 2018) and the stock market indices studied. Most of the indices (with the exception of S&P 500, Dow Jones Industrial 30 and DAX30) are from European Union countries (except Switzerland). Some are neighbouring countries (investor behaviour should be similar) but the common feature is that they have not yet adopted the euro (transactions in the national currency, foreign investors thus assuming, besides market risk and foreign exchange risk in the moment of making investments on the capital markets of these countries).

Another element of originality is that the stock indexes analysed are less well-researched in the previous studies, the reason for the exclusion being that some of the capital markets are small size and thus the interest of the foreign investors is lower.

2. Literature review

Forecasting volatility (volatility perceived as a source of risk by investors) has constituted a subject of study for the international scientific community. In time, a lot of models of volatility study have been proposed and tested for various time series and different frequencies.

The first model of volatility estimation was Black and Scholes (1975, pp. 307-324) for implicit volatility in options, followed by the ARMA model proposed by Box and Jenkins (1976) used to study the volatility of financial assets. These models were based on the assumption that the price series of the financial assets have a constant variance, the hypothesis that proved to be erroneous. Previous models cannot capture the stylized facts (Cont, 2001) of the financial returns such as: volatility clustering, leptokurtosis, leverage effect, fat tail, etc.

The ARCH (Autoregressive Conditional Heteroscedasticity) is a model proposed by Engle (1982), in which the variation depends on the previous patch errors, seemed to solve the above problems. The basis of the model was the empirical observations of the change in time of volatility and the fact that it depends on its previous values. But there was another problem, that the coefficients of the ARCH model are hard to estimate. Four years later, Bollerslev (1986), proposed an improved form of ARCH, namely GARCH (Generalized Autoregressive Conditional Heteroscedasticity). Empirical observations have shown that financial time series do not usually have a normal distribution (assuming skewness 0 and kurtosis 3) and rather a leptocurtotic one.

These observations underlie the leverage effect (the effect that news has on volatility) first presented by Black (1976). It has been noticed that negative news has a stronger impact on volatility than positive ones. The GARCH (1.1) fails to capture the leverage effect and so it was necessary to develop extensions of this model such as EGARCH, TGARCH, GARCH-M, etc. In the case of financial investments, assuming an increased risk is associated with a high expected return.

To capture the relationship between the expected return and the associated risk of a financial asset, Engle, Lilien and Robins (1987) expanded the GARCH model, introducing a new term, the conditioned volatility, in the mean equation of the classical model. All of these models have been tested on different markets and financial assets over different periods of time and on different frequencies (daily, weekly, monthly) and their conclusions varying. Akigray (1989, pp. 55-80) tested ARCH (2), EMWA and GARCH (1.1) to identify the time series properties of US expected stock return. The conclusion of the study was that GARCH (1,1) is the most appropriate model. Pagan and Schwert (1990, pp. 267-290) concluded that the EGARCH model is more performing than nonparametric models.

Cao and Tsay (1992, pp 165-185) supported the EGARCH model providing the best predictions for low capitalization shares. The study by Sill (1993, pp. 15-27) concluded that the volatility of the S&P500 index is higher in times of recession than in economic expansion and that spreads between corporate bond rates and government bonds predict future stock market volatility. Donaldson and Kamastra (1997, pp. 17-46) found that the persistence of volatility effects in European and North American markets is lower relative to Japanese market. Franses and Djik (1998, pp. 229-235) compared volatility predictions of QGARCH (1.1), GJR-GARCH (1,1), GARCH

(1,1) and Random Walk for stock indices in Spain, Germany, Italy, Netherlands and Sweden.

Nam, Pyun and Aruza (2002, pp. 563-588) applied the GARCH-M model for US stock indices during the period 1926-1997. They concluded that negative returns on average reverted more rapidly to long-term mean than positive returns.

Harq et al. (2004, pp 19-42) tested Random Walk, ARMA and GARCH-M for ten African and Middle East markets.

Selcuk (2004, pp. 867-874) surprised the persistence of volatility effect in emerging markets. Caiado (2004, pp. 3-21) investigated mean reversion behaviour for the Portuguese market (using the PSI20 index) and found that the mean reversion is recorded for low frequency and not for high frequency data.

Lupu (2005) demonstrated that GARCH model captures the characteristics of the Romanian capital market. Two years later, Lupu and Lupu (2007) used the EGARCH model to investigate the same capital market.

Rizwan and Khan (2007, pp. 362-375) have surprised the phenomenon of volatility clustering on the Pakistan market. Magnus and Fosu (2006, pp. 2042-2048) found a high level of persistence for the Ghana capital market.

Tudor (2008, pp.183-2008) tested the GARCH and GARCH models for the main indices of the American and Romanian financial markets. The GARCM-M model performed better and revealed the correlation between volatility and expected returns on both markets.

Panait and Slăvescu (2012) investigated the applicability of GARCH-M (1,1) on the Romanian capital market (1997-2012) for low and high-frequency data. The results of the study were in line with those of Caido (2004). In the Panait and Slăvescu' study, the mean reverting was ascertained for low frequency data (weekly and monthly) and less for high frequency data (daily). But, GARCH-M (1.1) "failed to confirm (...) the theoretical hypothesis that an increase in volatility leads to a rise in future returns, mainly because the variance" (Panait and Slăvescu, 2012, pp. 55).

The study of the Romanian capital market was continued by Miron and Tudor (2010). The paper focused on asymmetric GARCH, EGARCH, PGARCH, TGARCH) with a daily data frequency. For the model errors were used different distributions (normal distribution t, GED distribution and t student). The conclusion was that the

EGARCH (with Student and GED errors distributions) best surprised the characteristics of returns for Romanian capital market.

EGARCH model was best evaluated in the estimation of exchange rate volatility and stock indices and by other authors such as Lee (1991), Heyen and Kat (1994).

We will continue to investigate the applicability of the GARCH and GARCH-M models for a more recent period of time, March 2013 - January 2018, and for a number of stock indices little found in previous studies.

The main objective is to discover the current characteristics of capital markets, which would be a useful tool for all investors to substantiate the investment strategy.

3. Data and research methodology

As mentioned above, in our research we included fourteen stock indices: twelve in Europe and two of the main US stock indices (Dow Jones Industrial Average and S&P 500).

The indices and the number of daily observations for each index can be found below (Table 1).

Table 1

The stock indices included in the study

Symbol	Index name	Country	Nr obs.
DAX	Deutscher Aktien IndeX 30	Germany	1248
GSPC	The Standard & Poor's 500	US	1240
DJI	The Dow Jones Industrial Average	US	1240
BET	Bucharest Exchange Trading	Romania	1233
BETBK	Bucharest Exchange Trading Benchmark Iindex	Romania	1233
BGTR30	*BG TR30 Index*	Bulgaria	1215
BUX	Budapest Stock Exchange Index	Hungary	1226
CROBEX	The Croatia Stock Market	Croatia	1226
FTSE	The Financial Times Stock Exchange 100 Index	England	1244
KAX	KAX All-Share Index	Denmark	1228
OMX30	The OMX Stockholm 30	Sweden	1236
PX	Prague Stock Exchange *Index*	Czech Rep.	1229
WIG30	Warsaw Stock Exchange *Index*	Poland	1227
SMI	The Swiss Market Index	Switzerland	1235

Source: Yahoo Finance and Investing.com. Calculations by the authors

The time series of the fourteen stock indices are adjusted to corporate events (dividends, capital increases, consolidations, etc.) according to their calculation methodology.

Considering that the purpose of our analysis was not to study the correlation between these stock indices, there was no need for the perfect chronological synchronization of the data series analysed.

The study period is March 2013-January 2018, the frequency of the data is daily.

Price ranges were obtained from https://finance.yahoo.com for US stock indices and www.investing.com for European stock indices.

With the exception of DAX30 (which is expressed in EUR), all other stock indices are expressed in the national currency of those countries. In Figure 1 of the annexes we have graphical representations of the initial time series included in the study.

The price series were subsequently transformed into series of logarithmic returns, resulting in a database of fourteen logarithmic returns series.

Table 2

Descriptive statistics for the returns series

	Mean	**Maxim**	**Minim**	**Std. Dev.**	**Skew**	**Kurtosis**	**Jarque-Bera**	**P-val**
DAX	**4.4814**	479.69	-699.8	**115.15**	-0.344	5.426	320.67	0
SPX	**1.0635**	70.02	-72.36	**13.79**	-0.460	5.842	460.81	0
DJI	**8.7314**	619.07	-610.32	**125.63**	-0.331	5.204	267.05	0
BET	**1.8882**	213.73	-461.58	**53.19**	-0.887	10.907	3309.3	0
BET-BK	**0.4559**	43.47	-81.81	**9.0251**	-1.122	13.522	5836.2	0
BGTR30	0.2377	20.62	-19.79	2.7214	-0.186	11.574	3713.8	0
BUX	**16.279**	1281.72	-1341.9	**240.84**	-0.219	5.4846	321.01	0
CROBEX	-0.084	47.68	-62	10.32	-0.544	8.2089	1427.8	0
FTSE	0.9642	219.67	-288.78	55.82	-0.170	4.9700	201.55	0
KAX	0.5853	52.07	-55.05	10.50	-0.413	6.2901	580.17	0
OMX30	0.3138	55.270	-114.63	14.58	-0.492	7.3030	982.51	0
PX	0.0797	43.28	-45.68	8.39	-0.398	5.4003	322.56	0
WIG30	0.3536	79.25	-141.65	24.83	-0.312	5.3872	307.07	0
SMI	**1.6085**	289.9	-797.59	**79.23**	-1.178	13.638	5985.9	0

Source: Yahoo Finance and Investing.com, calculations by the authors

From Table 2 we draw the next conclusions:

All indices had an upward trend, except for the Croatian stock exchange index. For all the time series the value of standard deviation is larger than the mean values.

All data series present negative asymmetry, excess kurtosis and fat tail, indicating leptocurtotic distributions. The deviation from normality is more pronounced in the case of the SMI index (in Switzerland), with a skewness (-1.178) and a kurtosis (13.6385), values being far from those of the gaussian distribution (skewness 0 and kurtosis 3). The same characteristic can be observed for the Romanian indices: BET-BK and BET.

None of the time series are normally distributed, as proven by values for the Jarque-Bera tests (Table no. 2).

We continued to perform tests to determine heteroscedasticity and volatility clustering. The analysed series of returns show the phenomenon of volatility clustering, a phenomenon considered to be a consequence of the leptokurtotic distribution. This is the tendency of very high or very low volatile volatility periods to group together. The explanation for this phenomenon is that abnormally large shocks occurring during the current period will cause an immediate increase in volatility, and this will also rise in the next period, depending on investors' perception of the intensity of these shocks.

For the investigation of heteroscedasticity, we calculated the autocorrelation (AC), the partial autocorrelation (PAC) and applied the Q test (Ljung-Box statistic), the results being centralized below (Table 3). In our calculations, we used a 20 period lags. We can see that most data series present serial correlation till the 20-th lag (the Q test being significant at 10%), thus confirming the presence of heteroscedasticity. We also have three exceptions: DJI, BET and BET-BK for which the probability associated with the Q test does not allow us to reject the null hypothesis, the lack of the serial correlation till the 20-th lag.

Table 3
Estimation of autocorrelation, partial autocorrelation and Q-test with 20 lag

	lag	AC	PAC	Q-Stat	P-val.
DAX	20	0.046	0.053	35.628	0.017
SPX	20	0.058	0.057	31.148	0.053
DJI	20	0.060	0.053	21.104	0.391
BET	20	-0.038	-0.041	17.697	0.607
BETBK	20	-0.023	-0.027	18.318	0.566
BGTR30	20	0.031	0.028	36.911	0.012
BUX	20	-0.022	-0.024	33.036	0.033
CROBEX	20	0.028	-0.002	126.94	0.000
FTSE	20	0.036	0.040	34.994	0.020
KAX	20	-0.021	-0.030	29.279	0.082
OMX30	20	0.055	0.049	53.515	0.000
PX	20	-0.055	-0.053	32.887	0.035
WIG30	20	0.005	0.015	31.058	0.054
SMI	20	-0.009	-0.013	37.493	0.010

Source : Yahoo Finance and Investing.com, calculations by the authors

In conclusion, we found heteroscedasticity in returns for only eleven of the fourteen series studied. As heteroscedasticity is a precondition for applying GARCH models, it is possible we cannot calibrate these models for the three series of returns (DJI, BET and BET-BK). After we discovered the presence of the phenomenon of volatility clustering and heteroscedasticity, we passed to the estimation of the parameters of GARCH (1.1) and GARCH-M (1.1) for all fourteen datasets.

As previously mentioned, the GARCH model (1.1) was proposed by Bollerslev (1986) and has two equations, one for the mean and one for the variance of the time series presented below.

The mean equation: $Ri=\mu+\varepsilon_i$ (1)

The variance equation: $\sigma i^2=\omega+\alpha\varepsilon^2{}_{i-1}+\beta\sigma^2{}_{i-1}$ (2)

Where: ω is the mean, $\varepsilon^2{}_{i-1}$ is the term ARCH (the last volatility information measured as lag of the squared residuals of the mean equation), $\sigma^2{}_{i-1}$ is the term GARCH (the forecast variance of the previous period). We observe that the conditional variance (the variance of the next period calculated on the basis of the previous values) is a function of three variables: the mean (ω), the term ARCH and the term GARCH. The persistence of conditional volatility is given by the sum of ARCH and GARCH coefficients and it must be subunit ($\alpha+\beta<1$). This is an essential condition for a mean reverting process.

The GARCH-M (1.1) proposed by Engle, Lilien and Robins (1987) is an extension of the GARCH (1.1) model and has the following equations:

The mean equation: $Ri=\mu+\beta i\sigma i^2+\varepsilon_i$ (3)

The variance equation: $\sigma i^2=\omega+\alpha\varepsilon^2_{i-1}+\beta\sigma^2_{i-1}$ (4)

Unlike the initial model, GARCH (1.1) GARCH-M (1.1) has the term β_i representing the volatility of the analysed assets in the mean equation. It should capture the positive relationship between the assumed risk and the expected future return of this placement. One of our study objectives was to investigate the existence of this relationship for our set of stock indices between March 2013 and January 2018.

4. Results and interpretations

In table 4 (found in Annexes) are presented the values for the coefficients: ω, α and β of the GARCH (1.1) model. In estimating model for each of the fourteen series we started from the assumption that the errors are normally distributed. Analysing the data presented in the table we can conclude that all coefficients of the variance equation (ω, α and β) are statistically significant for all data series at a high confidence level, 99%. It had high values for z-statistical and low p-value. The estimated coefficients of the model fulfil the condition that $\alpha + \beta<1$, a condition necessary for the process to be mean reverting. If $\alpha + \beta> 1$, the process would be an explosive one, and the modelling of the data series would have to be done with another GARCH model (the IGARCH model).

We can conclude that the most time series (except WIG30) are mean reverting. To investigate the return to average volatility behaviour (more precisely the persistence of volatility) we calculated the sum of the coefficients α and β (Table 5).

Table 5

The persistence value in the GARCH (1,1)

Indices	Persistence
DAX	0.986006
SPX	0.915459
DJI	0.878765
BET	0.906898
BET-BK	0.805244
BGTR30	0.950867
BUX	0.938798
CROBEX	0.932255
FTSE	0.931947
KAX	0.983791
OMX30	0.975901
PX	0.953836
WIG30	-0.18871
SMI	0.973519

Source: calculations by the authors

It can be noticed that the conditioned volatility of returns for BET-BK tend to revert fastest to the long-term mean, followed by Dow Jones (0.8787) and BET (0.9068). We note that the conditioned volatility of the Romanian indices tends to revert to the mean faster than the other indices included in the study. We proceeded to evaluate the relevance of the GARCH (1.1) through statistical tests on standardized residuals of the model. The GARCH (1.1) is correctly specified if the standardized residuals will no longer show serial correlation, heteroscedasticity or any other linear dependence.

Table 6

Tests for residuals of GARCH (1.1) model

	Standardized residuals			**Squared standardized residuals**			**ARCH-LM (p-val.)**	**Jarque -Bera (p-val.)**
	AC	**PAC**	**Q-stat**	**AC**	**PAC**	**Q-stat**		
DAX	0.051	0.052	28.97 (0.88)	-0.003	-0.007	6.90 (0.99)	0.33 (0.99)	157.9 (0.00)
SPX	0.041	0.039	20.69 (0.41)	-0.014	-0.016	5.97 (0.99)	0.29 (0.99)	505.9 (0.00)
DJI	0.048	0.041	17.09 (0.64)	-0.011	-0.017	10.61 (0.95)	0.53 (0.95)	150.2 (0.00)

	Standardized residuals			Squared standardized residuals			ARCH-LM (p-val.)	Jarque -Bera (p-val.)
	AC	PAC	Q-stat	AC	PAC	Q-stat		
BET	-0.026	-0.026	18.73 (0.53)	0.021	0.023	6.20 (0.99)	0.31 (0.99)	1505.9 (0.00)
BET-BK	-0.015	-0.013	15.18 (0.76)	-0.001	-0.002	8.87 (0.98)	0.42 (0.98)	1665.7 (0.00)
BGTR30	0.066	0.056	37.35 (0.01)	-0.015	-0.021	16.27 (0.7)	0.79 (0.72)	316.3 (0.00)
BUX	-0.019	-0.026	29.64 (0.07)	0.003	0.003	10.90 (0.94)	0.54 (0.94)	207.0 (0.00)
CROBEX	0.042	0.032	30.06 (0.06)	0.029	0.027	13.07 (0.87)	0.62 (0.89)	480.6 (0.00)
FTSE	0.012	0.015	20.18 (0.44)	0.004	0.002	6.05 (0.99)	0.29 (0.99)	91.4 (0.00)
KAX	-0.027	-0.027	15.51 (0.74)	0.007	0.003	12.63 (0.89)	0.55 (0.94)	100.5 (0.00)
OMX30	0.06	0.053	30.37 (0.06)	0.014	0.013	11.63 (0.92)	0.59 (0.92)	105.9 (0.00)
PX	-0.039	-0.035	24.62 (0.21)	-0.023	-0.028	15.21 (0.76)	0.78 (0.73)	273.7 (0.00)
DWIG30	0.009	0.015	23.52 (0.26)	-0.017	-0.022	23.76 (0.25)	1.12 (0.31)	136.0 (0.00)
SMI	0.009	0.009	24.63 (0.21)	0.005	0.006	12.56 (0.89)	0.62 (0.89)	729.0 (0.00)

Source: calculations by the authors

To verify the existence of serial correlations in standardized residuals, we investigated autocorrelation (AC function), partial correlation (PAC function) and applied the Ljung-Q-Box test till the 20-th lag.

We applied the ARCH test (using Lagrange multiplier) to investigate whether we still have ARCH effects in residuals.

The model is appropriate if we notice the lack of these effects. We also applied the Jarque-Bera test to see if the residuals are normally distributed or not. The results of these tests are summarized in Table 6.

The data (presented in Table 6) leads us to conclude that the GARCH (1.1) model is relevant to the analysed financial data series. Simple standardized and squared standardized residuals are not auto-correlated as shown results of AC, PAC, and Q tests.

The ARCH-LM test tells us that there are no ARCH effects in residuals, so the GARCH (1.1) is correctly specified.

The Jarque-Bera test indicates that residuals are not normally distributed, but this feature is often found in the residuals of the appropriate models for the financial time series.

The results of applying the GARCH-M model (1.1) on the same series of financial data are presented in Table 7 of the Annexes. In the table we find the values of the coefficients β, ω, α and β of this model. We started from the assumption that the errors are normally distributed.

The data presented in the table leads us to the following conclusions:

1) The coefficients of the variant equation (ω, α and β) are statistically significant for most time series at the 99% confidence level, with one exception WIG30, whose GARCH coefficient is statistically significant at the 95% confidence level. We have high values for z-statistical and p-low value;
2) Estimated coefficients of the model fulfils the requirement that α + β <1, an essential condition for a mean reverting process. We can conclude that conditional volatilities are mean reverting for all the returns series (except the data tome for WIG30).
3) Unfortunately, the β1 coefficient of the variance term in the mean equation is positive and statistically significant (at the 90% confidence level) only for six series of financial data, respectively SPX, DJIA, FTSE (major indices) and BET, BET- BK and BUX.

Interesting to note, the last three indices are representative indices for the capital markets in Romania and Hungary, neighbouring countries. For the BET index, the results are in line with most of the previous studies and in contradiction with one conducted over the period 1997-2012 showing that the application of the GARCH-M (1.1) model failed to eliminate the ARCH effects of the standardized residual series for daily frequency data.

In conclusion, other GARCH models should be better for modelling and forecasting volatility for the remaining eight times series, for which GARCH-M (1,1) was not the appropriate model. In Table 8 we can see that the conditioned volatility for the BET-BK returns has the fastest mean reverting tendency followed by the Dow Jones and BET returns series.

Table 8

The persistence value in the GARCH-M (111)

Indices	Persistence
DAX	0.985299
SPX	0.911508
DJI	0.881048
BET	0.904979
BET-BK	0.782398
BGTR30	0.950732
BUX	0.934464
CROBEX	0.93522
FTSE	0.935373
KAX	0.979206
OMX30	0.975613
PX	0.954027
WIG30	-0.20512
SMI	0.972802

Source: calculations by the authors

We proceeded to evaluate the relevance of the GARCH-M (1.1) through statistical tests on standardized residuals of the model. GARCH-M (1.1) is correctly specified if the standardized residuals will no longer show serial correlation, heteroscedasticity or any other linear dependence.

To verify the existence of serial correlations in standardized residuals, we investigated autocorrelation (AC function), partial correlation (PAC function) and applied the Ljung-Box test till the 20-th lag. We applied the ARCH test (using Lagrange multiplier) to investigate whether we still have ARCH effects in residuals. The model is appropriate if we notice the lack of these effects. We also applied the Jarque-Bera test to see if the residuals are normally distributed or not. The results are summarized below (Table 9).

Table 9

Tests for residuals of GARCH-M (1.1) model

	Standardized residuals			**Squared standardized residuals**			**ARCH-LM (p-val.)**	**Jarque-Bera (p-val.)**
	AC	**PAC**	**Q-stat**	**AC**	**PAC**	**Q-stat**		
DAX	0.05	0.053	29.02 (0.08)	-0.003	-0.006	6.76 (0.99)	0.33 (0.99)	159.59 (0.00)
SPX	0.045	0.043	19.77 (0.47)	-0.009	-0.011	5.51 (0.99)	0.27 (0.99)	487.92 (0.00)
DJI	0.05	0.043	18.09 (0.58)	-0.006	-0.012	10.31 (0.96)	0.51 (0.96)	156.55 (0.00)
BET	-0.028	-0.028	21.128 (0.39)	0.021	0.022	6.09 (0.99)	0.30 (0.99)	1477.7 (0.00)
BET-BK	-0.017	-0.015	20.09 (0.45)	-0.002	-0.004	8.54 (0.98)	0.40 (0.99)	1691.2 (0.00)
BGTR30	0.066	0.056	37.47 (0.01)	-0.014	-0.02	16.55 (0.68)	0.84 (0.70)	307.84 (0.00)
BUX	-0.022	-0.029	29.07 (0.086)	0.002	0.002	11.10 (0.94)	0.54 (0.94)	215.5 (0.00)
CROBEX	0.041	0.032	29.59 (0.07)	0.028	0.026	12.78 (0.88)	0.61 (0.90)	469.55 (0.00)
FTSE	0.017	0.023	17.77 (0.60)	0.003	0.002	6.05(0.99)	0.29 (0.99)	91.46 (0.00)
KAX	-0.026	-0.026	14.99 (0.77)	0.011	0.006	12.37 (0.90)	0.55 (0.91)	103.89 (0.00)
OMX30	0.062	0.057	29.44 (0.08)	0.014	0.012	10.93 (0.94)	0.56 (0.93)	118,16 (0.00)
PX	-0.04	-0.036	25.03 (0.2)	-0.023	-0.028	15.13 (0.76)	0.77 (0.74)	269,33 (0.00)
WIG30	0.009	0.015	23.52 (0.26)	-0.018	-0.022	23.57 (0.26)	1.11 (0.32)	139.73 (0.00)
SMI	0.011	0.014	22.17 (0.33)	0.004	0.006	12.02 (0.91)	0.59 (0.91)	760,78 (0.00)

Source: calculations by the authors

From the test results presented in Table 9, we conclude that the GARCH-M (1.1) models used to characterize the volatility of financial time series for the fourteen indices are correctly specified.

The results of AC, PAC, and Q statistics test show that there is not statistically significant trace of autocorrelation in standardized residuals. The ARCH-LM test results also show that the model succeeded to eliminate all ARCH effects in residual series.

The Jarque-Bera test indicates that residuals are not normally distributed, but this feature is often found in the residuals of the appropriate models for the financial time series.

In conclusion, the GARCH-M (1,1) model is relevant to our financial data series, the model being able to eliminate the heteroscedasticity and ARCH effects of the daily series of standardized residuals, but the positive correlation between risk and expected return was confirmed for the less than half of the data series. It is recommended to test other models in the GARCH family, maybe asymmetric models, to study the volatility behaviour of these series of financial data in order to identify the most relevant model.

5. Conclusions

In this paper, we used GARCH (1.1) and GARCH-M (101) to characterize volatility on different European and American capital markets. The study included data for fourteen stock indices (from Europe and the US) during March 2013- January 2018, with a daily frequency of the price series. Most of the time series present the characteristics of volatility clustering and heteroscedasticity required to apply the GARCH model.

The GARCH (1.1) model proved to be appropriate for modelling the volatility of returns series, the coefficients of the ARCH and GARCH terms being statistically significant with one exception, the returns series for WIG30, where the GARCH coefficient in the conditional variance equation was negative.

The GARCH-M (1.1) model surprised the positive correlation between assumed risk and future returns for only six of the fourteen sets of financial data. Those were: main American stocks indices (SPX and DJIA), London Stock Exchange index (FTSE), Romanian stock indices (BET and BETBK) and the index of the Hungarian stock exchange (BUX). The results obtained in the case of BET are in line with those of the previous studies, but in contradiction with the study during the 1997-2012, which showed that the modelling of volatility through GARCH-M (1,1) failed to eliminate the effects of ARCH in the residuals series of the model. Conclusions for the BET-BK index are an element of originality for this paper, the index being a relatively new on the BSE and less found in other studies.

From the twelve European time series, only three of them could confirm the hypothesis that the increase in volatility leads to an increase in future returns. The three stock indices belong to neighbouring countries, Romania and Hungary. Perhaps this is a similarity of investors' financial behaviour for a geographic region

After application, both models succeeded to eliminate all traces of autocorrelation and ARCH effects in the standardized residuals series. All residual series continued to be not normally distributed, but this feature was often found in the case of the residuals of the models used to test the financial time series.

The coefficients of the two equations (mean and variance) were statistically significant and showed that conditional volatility tended to revert to the long- term mean, except for the WIG30 index. The coefficient of variance in the mean equation was statistically significant and positive for only six of the fourteen series of data. For these, we found a positive correlation between the risk assumed and the future return demanded by investors on this capital markets.

The persistence of volatility in mature capital markets was lower for US (the markets for which information with a potential negative impact had an insignificant and short-term influence, the strong upward trend being not interrupted by any negative news) and much higher for Germany (0.986) and the United Kingdom (0.93).

We also notice that the persistence of volatility was lower on US markets (in line with our expectations mentioned in the start of the study) compared witch one recorded in European markets. In the last case, volatility remained high, thus showing that on the European capital markets the shocks felt much stronger and their effects persisted for longer periods of time.

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APPENDIX

Table 4

Estimated values for GARCH (1.1) coefficients

	Variance eq.	Coefficient	Std. Error	z-Statistic	P-val.
DAX	ω	187.7588	59.30497	3.165988	0.0015
	α	0.061892	0.010537	5.873726	0.0000
	β	0.924114	0.013744	67.23961	0.0000
SPX	ω	16.24768	2.804355	5.793731	0.0000
	α	0.156303	0.023094	6.768035	0.0000
	β	0.759156	0.029648	25.60606	0.0000
DJI	ω	1916.611	333.5570	5.745978	0.0000
	α	0.184579	0.024221	7.620677	0.0000
	β	0.694186	0.040090	17.31570	0.0000
BET	ω	292.6598	57.73191	5.069290	0.0000
	α	0.147711	0.016914	8.733269	0.0000
	β	0.759187	0.028166	26.95392	0.0000
BET-BK	ω	16.17267	2.796304	5.783586	0.0000
	α	0.214069	0.017646	12.13145	0.0000
	β	0.591175	0.043070	13.72595	0.0000
BGTR30	ω	0.381840	0.073424	5.200501	0.0000
	α	0.144880	0.013796	10.50143	0.0000
	β	0.805987	0.021140	38.12686	0.0000
BUX	ω	3790.264	1062.313	3.567936	0.0004
	α	0.087997	0.015591	5.643895	0.0000
	β	0.850801	0.028489	29.86414	0.0000
CROBEX	ω	6.165254	1.214226	5.077518	0.0000
	α	0.076169	0.012519	6.084371	0.0000
	β	0.856086	0.021720	39.41454	0.0000
FTSE	ω	202.1501	40.81481	4.952861	0.0000
	α	0.122284	0.018530	6.599302	0.0000
	β	0.809663	0.026315	30.76857	0.0000
KAX	ω	2.032260	0.599750	3.388512	0.0007
	α	0.110842	0.014631	7.575998	0.0000
	β	0.872949	0.01646	53.00928	0.0000
OMX30	ω	5.311251	1.649986	3.218968	0.0013
	α	0.101532	0.016402	6.190254	0.0000
	β	0.874369	0.022308	39.19476	0.0000
PX	ω	3.297118	0.731536	4.507117	0.0000
	α	0.098451	0.012822	7.678009	0.0000
	β	0.855385	0.018564	46.07786	0.0000
WIG30	ω	727.1751	75.98581	9.569880	0.0000
	α	0.110934	0.022586	4.911624	0.0000
	β	-0.299641	0.101827	-2.942658	0.0033
SMI	ω	199.4564	46.89080	4.253635	0.0000
	α	0.130998	0.015702	8.342855	0.0000
	β	0.842521	0.019915	42.30590	0.0000

Source: calculations by the authors

Table 7

Estimated values for GARCH-M (1.1) coefficients

	Variance eq.	Coefficient	Std. Error	z-Statistic	P-val.
DAX	β_1	-0.009323	0.105018	-0.088773	0.9293
	ω	197.1702	61.00719	3.231917	0.0012
	α	0.064205	0.010842	5.922025	0.0000
	β	0.921094	0.014066	65.48344	0.0000
SPX	β_1	0.400867	0.120986	3.313319	0.0009
	ω	17.03805	2.975056	5.726967	0.0000
	α	0.167453	0.025904	6.464388	0.0000
	β	0.744055	0.033213	22.40283	0.0000
DJI	β_1	0.266370	0.125865	2.116309	0.0343
	ω	1880.606	331.3855	5.674982	0.0000
	α	0.184596	0.024638	7.492196	0.0000
	β	0.696452	0.040368	17.25259	0.0000
BET	β_1	0.293213	0.160875	1.822612	0.0684
	ω	297.6167	59.85748	4.972089	0.0000
	α	0.150351	0.016988	8.850520	0.0000
	β	0.754628	0.029117	25.91668	0.0000
BET-BK	β_1	0.309912	0.180409	1.717828	0.0858
	ω	17.91310	2.929159	6.115442	0.0000
	α	0.224950	0.018297	12.29438	0.0000
	β	0.557448	0.044995	12.38924	0.0000
BGTR30	β_1	-0.080370	0.110019	-0.730503	0.4651
	ω	0.381006	0.073885	5.156764	0.0000
	α	0.144905	0.014039	10.32130	0.0000
	β	0.805827	0.021154	38.09403	0.0000
BUX	β_1	0.300091	0.168632	1.779562	0.0751
	ω	4039.936	1102.252	3.665166	0.0002
	α	0.091377	0.016521	5.531090	0.0000
	β	0.843087	0.029753	28.33599	0.0000
CROBEX	β_1	-0.032104	0.167473	-0.191697	0.8480
	ω	5.927939	1.207571	4.908977	0.0000
	α	0.074998	0.012345	6.074956	0.0000
	β	0.860222	0.021567	39.88514	0.0000
	β_1	0.347581	0.127428	2.727665	0.0064
FTSE	ω	189.9959	40.43424	4.698886	0.0000
	α	0.116626	0.018024	6.470758	0.0000
	β	0.818747	0.026498	30.89884	0.0000
KAX	β_1	0.055858	0.096916	0.576358	0.5644
	ω	2.582926	0.766458	3.369949	0.0008
	α	0.127024	0.017182	7.392875	0.0000
	β	0.852182	0.019947	42.72267	0.0000
OMX30	β_1	0.124306	0.102908	1.207931	0.2271
	ω	5.383833	1.668031	3.227657	0.0012
	α	0.103449	0.017404	5.943945	0.0000
	β	0.872164	0.022760	38.32003	0.0000
PX	β_1	-0.073602	0.138742	-0.530497	0.5958
	ω	3.308343	0.745860	4.435606	0.0000
	α	0.099741	0.012972	7.688670	0.0000
	β	0.854286	0.018728	45.61651	0.0000

	Variance eq.	Coefficient	Std. Error	z-Statistic	P-val.
WIG30	β1	0.030778	0.270078	0.113958	0.9093
	ω	737.2732	100.5816	7.330102	0.0000
	α	0.106266	0.022565	4.709402	0.0000
	β	-0.311387	0.150800	-2.064893	0.0389
SMI	β_1	0.132316	0.111578	1.185870	0.2357
	ω	204.6108	48.02399	4.260595	0.0000
	α	0.133025	0.015912	8.360233	0.0000
	β	0.839777	0.020149	41.67866	0.0000

Source: calculations by the authors

Figure 1

Graphs of stock indices

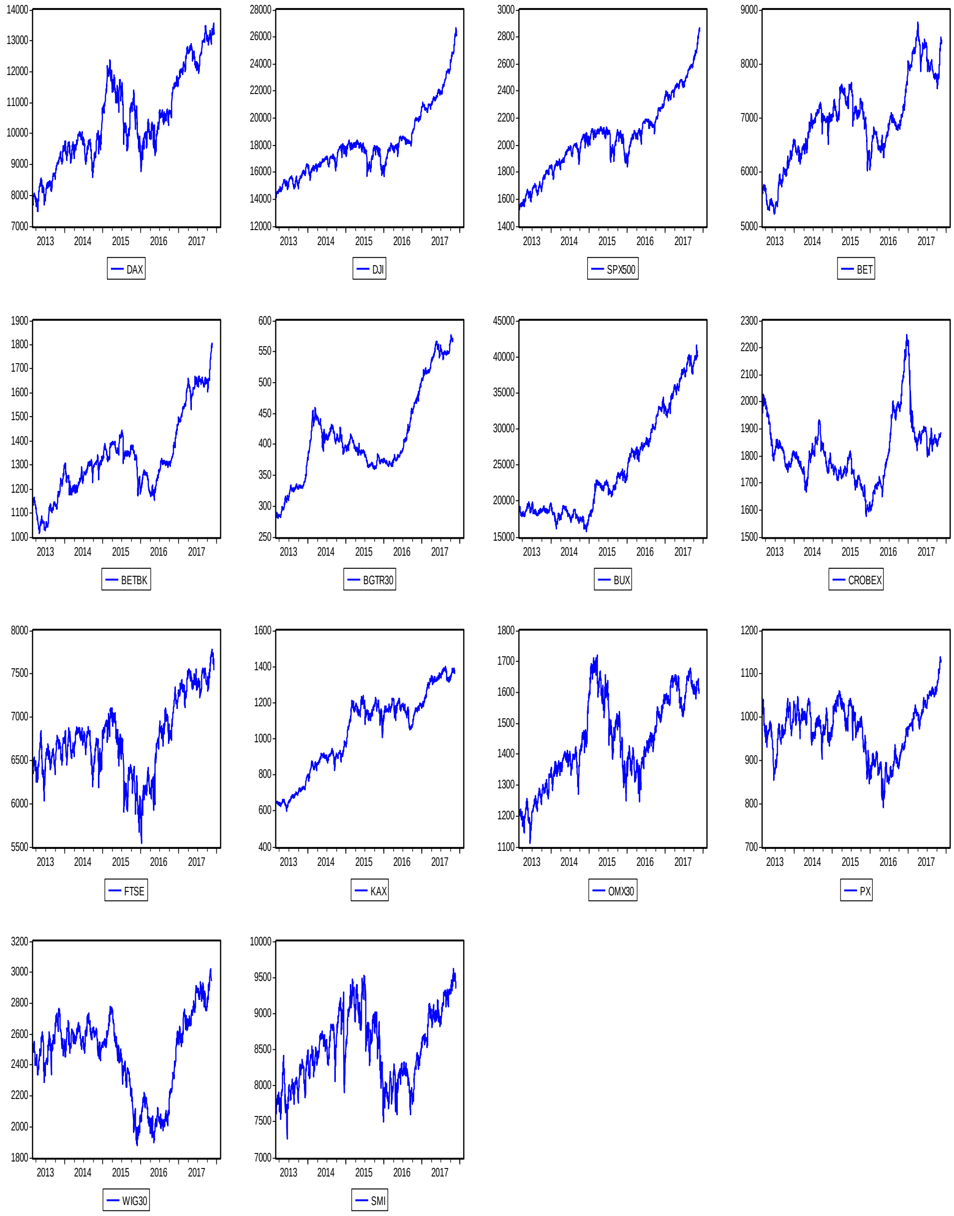

Figure 2

Series of logarithmic returns for all stock indices